Scalable Computing: Practice and Experience, ISSN 1895-1767, http://www.scpe.org
© 2024 SCPE. Volume 25, Issues 5, pp. 4335–4342, DOI 10.12694/scpe.v25i5.3178

ON SOFT STRONGLY B^*-COMPACTNESS AND SOFT STRONGLY B^*-CONNECTEDNESS IN SOFT TOPOLOGICAL SPACES

SAIF Z. HAMEED*, ABDELAZIZ E. RADWAN† AND ESSAM EL-SEIDY‡

Abstract. In this research article, we present a new class of soft compact spaces and soft Lindelöf spaces, we identify the idea of soft strongly b^*-compact and soft strongly b^*-Lindelöf spaces and we supply multiple interesting examples. As well as we mention that the inaugurated spaces are conserved under soft strongly b^*-irresolute mappings and we look into definite of results which connect an extensive soft topology with the showing soft spaces. As well as we inquiry the features and attributive of soft strongly b^*-connected spaces and discuss and identify its relationship with soft connectedness.

Key words: soft strongly b^*-closed set, soft strongly b^*-open set, soft strongly b^*-compact, soft strongly b^*-Lindelöf spaces, soft strongly b^*-connected space

1. Introduction and Preliminaries. Molodtsov [1] used an acceptable parametrization. In 1999, he introduced the soft set theorem's basic idea and disclosed the theorem's first result. He had many experimenters working on the proposal. Topology is eminent in colorful divaricate of mathematics. Therefore, Shabir and Naz [2] were the pioneers who introduced the concept of soft topological spaces. Kannan [3] assigned soft generalized closed and soft generalized open sets in soft topological spaces. Akdag and Ozkan ([4], [5]) presented a conception of soft $\alpha-$open, the soft $b-$open, and their respective continuous functions. Zorlutuna et al. inquiry soft interior point and soft neighbourhood and he first examined the compactness of soft topological spaces [6]. Connectedness [7] is an effective tool for topology introduced by Porter J. and Woods R.. Hussain [8] assigned and take a look at the features of soft connected space. Saif Z. et al. [9] introduced the soft $bc-$open set. The soft b^*-closed are introduced by Hameed, Saif Z. et al. [10]. Soft b^*-continuous functions, soft strongly b^*-closed and soft strongly b^*-continuous functions are studied by Hameed, Saif Z. et al. [11], [12].

In the present work, we define the soft strongly b^*-compact and soft strongly b^*-Lindelöf spaces. Also, we introduce the soft strongly b^*-connected spaces. The details of the properties, examples, and counterexamples that substantiate the concept are thoroughly discussed.

In this study, consider $\mathcal{W}$ as an initial universe and $P(\mathcal{W})$ as the power set of $\mathcal{W}$. In addition, $\check{E} \neq \phi)$ stands for the family of parameters that are being considered and $\phi \notin \wp \subseteq \check{E}$.

Definition 1.1. *[1] $(\Psi, \wp)$ is referred to be a soft set over $\mathcal{W}$ if Ψ is a map from $\wp$ to $P(\mathcal{W})$.*

Definition 1.2. *[13] The soft set $(\mathcal{S}, \wp) \in \mathcal{SS}(\mathcal{W}, \wp)$, where $S(\nabla) = \phi$, for every $\nabla \in \wp$ is stated A-null soft set of $\mathcal{SS}(\mathcal{W}, \wp)$ and symbolize by $\widetilde{\check{\phi}}$ The soft set $(\mathcal{S}, \wp) \in \mathcal{SS}(\mathcal{W}, \wp)$, where $S(\nabla) = \mathcal{W}$, for every $\nabla \in \wp$ is stated the A-absolute soft set of $\mathcal{SS}(\mathcal{W}, \wp)$ and symbolize by $\widetilde{\mathcal{W}}$.*

Definition 1.3. *[13] For two sets $(\Psi, \wp), (\mathcal{S}, \Theta) \in \mathcal{SS}(\mathcal{W}, \wp)$, then $(\Psi, \wp)$ is a soft subset of $(\mathcal{S}, \Theta)$ symbolize by $(\Psi, \wp) \subseteq (\mathcal{S}, \Theta)$, if*

1. *$\wp \subseteq \Theta$.*
2. *$\psi(\nabla) \subseteq S(\nabla), \forall \nabla \in \wp$.*

Then, $(\Psi, \wp)$ is stated to be a soft superset of $(\mathcal{S}, \Theta)$, if $(\mathcal{S}, \Theta)$ is a soft sub-set of $(\Psi, \wp)$, $(\mathcal{S}, \Theta) \subseteq (\Psi, \wp)$.

Definition 1.4. *[2] Let $(\Psi, \wp)$ be soft set over $\mathcal{W}$, $z \in \mathcal{W}$.that's what we call $z \in (\Psi, \wp)$, whenever $z \in \psi(\nabla)$ for all $\nabla \in \wp$. The soft set $(\Psi, \wp)$ over $\mathcal{W}$ such that $\psi(\nabla) = \{z\}, \forall \in \wp$ is stated singleton soft point and symbolize by $z_\wp$ or $(z, \wp)$.*

*Department of Mathematics, College of Education, Mustansiriyah University, Baghdad, Iraq (saif.zuhar.edbs@uomustansiriyah.edu.iq),
†Department of Mathematics, Faculty of Science, Ain Shams University, Cairo, Egypt (zezoradwan@yahoo.com).
‡Department of Mathematics, Faculty of Science, Ain Shams University, Cairo, Egypt (esam_elsedy@hotmail.com).

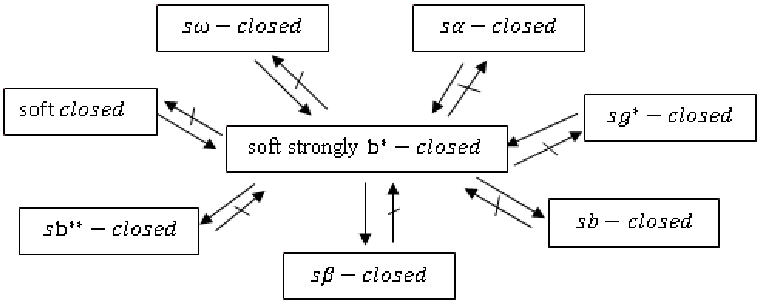

Fig. 1.1: Relationships of soft strongly b^*-closed

DEFINITION 1.5. *[2] Let $\mathcal{Q} \subseteq \mathcal{SS}(\mathcal{W}, \wp)$. Then $\mathcal{Q}$ is stated to be soft topological space ($\mathcal{STS}$) if*

1. *$\widetilde{\phi}$ and $\widetilde{\mathcal{W}}$ belong to $\mathcal{Q}$.*
2. *Arbitrary unions of members $\mathcal{Q}$ belongs to $\mathcal{Q}$.*
3. *Finite intersections of members $\mathcal{Q}$ belongs to $\mathcal{Q}$.*

It is symbolize by $(\mathcal{W}, \mathcal{Q}, \wp)$ (briefly $\mathcal{W}$).

DEFINITION 1.6. *[2] Let $(\mathcal{W}, \mathcal{Q}, \wp)$ be a $\mathcal{STS}$ over $\mathcal{W}$, then the organ of $\mathcal{Q}$ are stated to be soft open sets in $\mathcal{Q}$.*

DEFINITION 1.7. *[2] Let $(\mathcal{W}, \mathcal{Q}, \wp)$ be a $\mathcal{STS}$ over $\mathcal{W}$. A soft set $(\Psi, \wp)$ over $\mathcal{W}$ is stated to be a soft closed set in $\mathcal{W}$, if its relative complement $(\Psi, \wp)$ belongs to $\mathcal{Q}$.*

DEFINITION 1.8. *[6] Let $(\mathcal{W}, \mathcal{Q}, \wp)$ be a $\mathcal{STS}$ and $(\Psi, \wp) \in \mathcal{SS}(\mathcal{W}, \wp)$. Then*

1. *The soft closure of $(\Psi, \wp)$ is the soft set*
   $cl(\Psi, \wp) = \cap\{(\mathcal{S}, \wp) : (\mathcal{S}, \wp) \in \mathcal{Q}^c, (\psi, \wp) \subseteq (\mathcal{S}, \wp)\}$.

2. *The soft interior of $(\Psi, \wp)$ is the soft set*
   $int(\Psi, \wp) = \cup\{(\mathcal{S}, \wp) : (\mathcal{S}, \wp) \in \mathcal{Q}, (\mathcal{S}, \wp) \subseteq (\psi, \wp)\}$.

DEFINITION 1.9. *A soft set $(\Psi, \wp)$ of a $\mathcal{STS}$ $(\mathcal{W}, \mathcal{Q}, \wp)$ is stated to be*

1. *soft $\alpha-$open [4] if $(\Psi, \wp) \subset int(cl(int((\Psi, \wp))))$,*
2. *soft pre-open [14] if $(\Psi, \wp) \subset int(cl((\Psi, \wp)))$,*
3. *soft semi-open [15] if $(\Psi, \wp) \subset cl(int((\Psi, \wp)))$,*
4. *soft $\beta-$open [14] if $(\Psi, \wp) \subset cl(int(cl((\Psi, \wp))))$,*
5. *soft $b-$open [5] if $(\Psi, \wp) \subset int(cl((\Psi, \wp))) \cup cl(int((\Psi, \wp))))$.*

DEFINITION 1.10. *[16] A soft set $(\Psi, \wp)$ is called soft $\omega-$closed in a $\mathcal{STS}$ $(\mathcal{W}, \mathcal{Q}, \wp)$, if $cl(\Psi, \wp) \subseteq (\mathcal{S}, \wp)$ whenever $(\Psi, \wp) \subseteq (\mathcal{S}, \wp)$ and $(\mathcal{S}, \wp)$ is soft semi-open set in $\mathcal{W}$. The relative complement of $(\Psi, \wp)$ is called soft $\omega-$open in $\mathcal{W}$.*

DEFINITION 1.11. *[12] A soft set $(\Psi, \wp)$ of a $\mathcal{STS}$ $(\mathcal{W}, \mathcal{Q}, \wp)$ is called a soft strongly b^*-closed (briefly sSb^*-closed) if $cl(int(\Psi, \wp)) \subseteq (\mathcal{S}, \wp)$, whenever $(\Psi, \wp) \subset (\mathcal{S}, \wp)$ and $(\mathcal{S}, \wp)$ is $sb-$open. The complement of a sSb^*-closed set is stated to be sSb^*-open set.*

THEOREM 1.12. *[12] The following statements are correct:*

1. *Every soft open is sSb^*-open.*
2. *Every $s\alpha-$open is sSb^*-open.*
3. *Every sSb^*-open set is $sb-$open.*
4. *Every $s\omega-$open is sSb^*-open.*

DEFINITION 1.13. *[12] Let $(\mathcal{W}, \mathcal{Q}, \wp)$ be a $\mathcal{STS}$. a subset $(\Psi, \wp) \subseteq \mathcal{W}$ is called a soft strongly b^*-neighourhood (briefly sSb^*-nbd) of point $\nu \in \mathcal{W}$ if $\exists$ an sSb^*-open set $(\Psi, \wp)$ where $\nu \in \mathcal{W} \subseteq (\Psi, \wp)$.*

DEFINITION 1.14. *[12] Let $(\mathcal{O}, \wp) \in \mathcal{SS}(\mathcal{W}, \wp)$. Then*
$sSb^*int(\mathcal{O}, \wp) = \cup\{(\pounds, \wp) : (\pounds, \wp)$ *is a* sSb^*-*open set and* $(\pounds, \wp) \subset (\mathcal{O}, \wp)\}$.

DEFINITION 1.15. *[12] Let* $(\pounds, \wp) \in \mathcal{SS}(\mathcal{W}, \wp)$. *Then* $sSb^*cl(\pounds, \wp) = \cap\{(\Psi, \wp) : (\Psi, \wp)$ *is a* sSb^*-*closed set and* $(\pounds, \wp) \subset (\Psi, \wp)\}$.

DEFINITION 1.16. *[12] A soft mapping* $\Pi : \mathcal{W} \to \Sigma$, *from* $\mathcal{STS}$ $(\mathcal{W}, \mathcal{Q}, \wp)$ *into* $\mathcal{STS}$ (Σ, Ω, Θ), *is stated to be soft strongly* b^*-*continuous (briefly* sSb^*-*continuous) if the inverse image of every soft open set in* Σ *is a* sSb^*-*open set in* $\mathcal{W}$.

DEFINITION 1.17. *[12] A soft mapping* $\Pi : \mathcal{W} \to \Sigma$ *is stated to be soft strongly* b^*-*irresolute (briefly* sSb^*-*irresolute) if the inverse image of every* sSb^*-*closed set in* Σ *is a* sSb^*-*closed set in* $\mathcal{W}$.

For are details, we refer to [12], [6], [7].

2. Soft strongly b^*-compact spaces. In this section, We offer the conception of soft strongly b^*-compact and soft strongly b^*-Lindelöf spaces and The significant structural properties.

DEFINITION 2.1. *A collection* $\{(\psi_\epsilon, \wp) : \epsilon \in \zeta\}$ *of soft strongly* b^*-*open sets is called a soft strongly* b^*-*open cover of* $(\mathcal{W}, \mathcal{Q}, \wp)$, *if* $\widetilde{\mathcal{W}} = \bigcup_{\epsilon\in\zeta}(\psi_\epsilon, \wp)$.

DEFINITION 2.2. *A* $\mathcal{STS}$ $(\mathcal{W}, \mathcal{Q}, \wp)$ *is called soft strongly* b^*-*compact (resp. soft strongly* b^*-*Lindelöf), if each* sSb^*-*open cover of* $\widetilde{\mathcal{W}}$ *has a finite (resp. countable) soft subcover of* $\widetilde{\mathcal{W}}$.

DEFINITION 2.3. *A soft subset* $(, \wp)$ *of a* $\mathcal{STS}$ $(\mathcal{W}, \mathcal{Q}, \wp)$ *is called soft strongly* b^*-*compact in* $\mathcal{W}$ *determined by for every collection* $\{(\psi_\epsilon, \wp) : \epsilon \in \zeta\}$ *of soft strongly* b^*-*open sets of* $\mathcal{W}$ *where* $(, \wp) \subset \cup\{(\psi_\epsilon, \wp) : \epsilon \in \zeta\}$ $\exists$ *finite subset* ζ_0 *of* ζ *where* $(, \wp) \subset \cup\{(\psi_\epsilon, \wp) : \epsilon \in \zeta_0\}$

DEFINITION 2.4. *A* $\mathcal{STS}$ $(\mathcal{W}, \mathcal{Q}, \wp)$ *is called soft strongly* b^*-*space if every* sSb^*-*open set of* $\mathcal{W}$ *is soft open set in* $\mathcal{W}$.

COROLLARY 2.5. *If* $\mathcal{STS}$ $(\mathcal{W}, \mathcal{Q}, \wp)$ *is a* sSb^*-*compact space and soft strongly* b^*-*space, then* $\mathcal{W}$ *is soft compact space.*

Proof. Assume that $\{(\psi_\epsilon, \wp) : \epsilon \in \zeta\}$ be soft open cover of $\mathcal{W}$. For each soft open set is sSb^*-open set, $\{(\psi_\epsilon, \wp) : \epsilon \in \zeta\}$ is sSb^*-open cover of $\mathcal{W}$. For $\mathcal{W}$ is sSb^*-compact space and sSb^*-space, $\exists$ finite subset ζ_0 of ζ where $\mathcal{W} \subset \{(\psi_\epsilon, \wp) : \epsilon \in \zeta\}$. Therefore, $\mathcal{W}$ is soft compact space. □

COROLLARY 2.6. *If* $\Pi : \mathcal{W} \to \Sigma$ *is a* sSb^*-*continuous function and* sSb^*-*space, then* Π *is soft continuous function.*

Proof. Assume $\{(\psi_\epsilon, \wp) : \epsilon \in \zeta\}$ be soft open set of Σ. whereas Π is sSb^*-continuous, $\{\Pi^{-1}((\psi_\epsilon, \wp)) : \epsilon \in \zeta\}$ is sSb^*-open set of $\mathcal{W}$ and whereas $\mathcal{W}$ is sSb^*-space, $\{\Pi^{-1}((\psi_\epsilon, \wp)) : \epsilon \in \zeta\}$ forms soft open set of $\mathcal{W}$. Thus, Π is soft continuous. □

COROLLARY 2.7. *Assume* $(\mathcal{W}, \mathcal{Q}, \wp)$ *be* $\mathcal{STS}$. *If* $(\mathcal{W}, \mathcal{Q}_\nabla)$ *is a* sSb^*-*compact space, for each* $\nabla \in \wp$, *then* $(\mathcal{W}, \mathcal{Q}, \wp)$ *is a* sSb^*-*compact space.*

Proof. Assume that $\wp = \{\nabla_1, \nabla_1, ..., \nabla_n\}$ be a set of parameter and $(\mathcal{W}, \mathcal{Q}_\nabla)$ is sSb^*-compact space, for each $\epsilon = \overline{1, n}$. Suppose $\{(\psi_\epsilon, \wp) : \epsilon \in \zeta\}$ be sSb^*-open cover of $\mathcal{W}$. Since $\cup_{\epsilon\in\zeta}(\psi_\epsilon, \wp)(\nabla) = \widetilde{\mathcal{W}}$, for each $\nabla \in \wp$, and $(\mathcal{W}, \mathcal{Q}_\nabla)$ is a sSb^*-compact, $\exists$ finite subset ζ_0 of ζ where $\cup_{\epsilon\in\zeta_0}(\psi_\epsilon, \wp)(\nabla) = \widetilde{\mathcal{W}}$. Hence, $\{(\psi_\epsilon, \wp) : \epsilon \in \zeta_0\}$ is a finite subcover of $\{(\psi_\epsilon, \wp) : \epsilon \in \zeta\}$. Hence, $(\mathcal{W}, \mathcal{Q}, \wp)$ is a sSb^*-compact space. □

COROLLARY 2.8. *Every* sSb^*-*compact (resp.* sSb^*-*Lindelöf) space is soft compact (resp. soft Lindelöf)).*

In the next example, indicates that the inclusions of the Corollary 2.8 is not necessarily correct.

Example 1. Consider $\wp = Q^c$ is the set of irrational numbers. Let $\mathcal{Q} = \{\widetilde{\tilde{\phi}}, \widetilde{\mathcal{W}}, (, \wp)$ when $(\nabla) = \{1\}$, $\forall\nabla \in \wp\}$ be a $\mathcal{STS}$ on $\mathcal{W} = \{1, 2\}$. clearly, $(\mathcal{W}, \mathcal{Q}, \wp)$ is soft compact. furthermore, a family $\{(\delta, \Theta) : \delta(\upsilon) = \{1\}$, $\forall\upsilon \neq \nabla\}$ is a sSb^*- open cover of $\widetilde{\mathcal{W}}$. For has not a soft countable subcover of $\widetilde{\mathcal{W}}$. Thus, $(\mathcal{W}, \mathcal{Q}, \wp)$ is not a sSb^*-Lindelöf space.

THEOREM 2.9. *Every* sSb^*-*compact space is a* sSb^*-*Lindelöf.*

Proof. Clear. □

In the next example, indicates that the inclusions of the Theorem 2.9 and Figure 2.1 is not necessarily correct.

Example 2. Let $\mathcal{Q} = \{\widetilde{\tilde{\phi}}, \widetilde{\aleph}, (\delta, \wp)\}$ and $\wp = \{\nabla_1, \nabla_1, ..., \nabla_n\}$. such that $\delta(\nabla) = \{1\}$, $\forall\nabla \in \wp\}$ be a $\mathcal{STS}$ on the set of natural numbers $\aleph$. Since $\wp$ and $\aleph$ are soft countable, then $(\aleph, \mathcal{Q}, \wp)$ is a sSb^*-Lindelöf. furthermore, a family $\{(\mathcal{S}, \Theta) : \mathcal{S}(\upsilon) = \{1, x\}$, for each $\upsilon \in \Theta, x \in \aleph\}$ is a sSb^*-open cover of $\widetilde{\aleph}$. For has not soft finite subcover of $\widetilde{\aleph}$. Thus, $(\aleph, \mathcal{Q}, \wp)$ is not a sSb^*-compact.

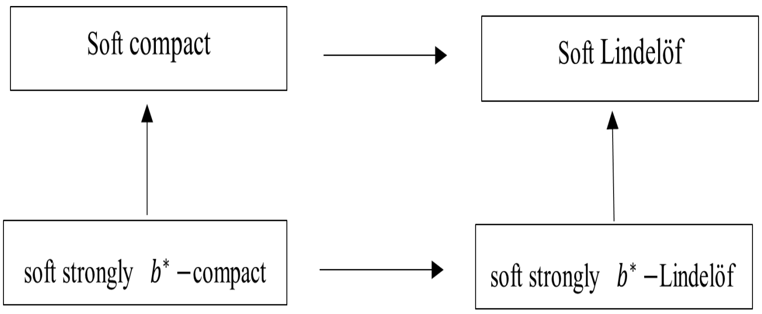

Fig. 2.1: Relationships

Theorem 2.10. *The soft union of two sSb^*-compact (resp. sSb^*-Lindelöf) sets is sSb^*-compact (resp. sSb^*-Lindelöf).*

Proof. Let $(\psi,\wp)$ and $(,\wp)$ be two sSb^*-compact sets. Assume that $\{(\psi_\epsilon,\wp):\epsilon\in\zeta\}$ is a sSb^*-open cover of $(\psi,\wp)\cup(,\wp)$. Then, $\{(\psi_\epsilon,\wp):\epsilon\in\zeta\}$ is a sSb^*-open cover of $(\psi,\wp)$ and $(,\wp)$. Since $(\psi,\wp)$ and $(,\wp)$ are sSb^*-compact, there exist finite subfamilies ζ_0 and ζ_1 of ζ such that $(\psi,\wp)\subseteq\{(\psi_\epsilon,\wp):\epsilon\in\zeta_0\}$ and $(,\wp)\subseteq\{(\psi_\epsilon,\wp):\epsilon\in\zeta_1\}$. Hence, $(\psi,\wp)\cup(,\wp)\subseteq(\cup\{(\psi_\epsilon,\wp):\epsilon\in\zeta_0\})\cup(\cup\{(\psi_\epsilon,\wp):\epsilon\in\zeta_1\})$. It follows that, $(\psi,\wp)\cup(,\wp)\subseteq\cup\{(\psi_\epsilon,\wp):\epsilon\in\zeta_0\cup\zeta_1\}$. Thus, $(\psi,\wp)\cup(,\wp)$ is a sSb^*-compact.
The proof of the case of sSb^*-Lindelöfness is similar. □

Theorem 2.11. *Every sSb^*-closed subset $(\pounds,\wp)$ of sSb^*-compact $(\mathcal{W},\mathcal{Q},\wp)$ is a sSb^*-compact.*

Proof. Assume that $(\pounds,\wp)$ be a sSb^*-closed subset of sSb^*-compact space $(\mathcal{W},\mathcal{Q},\wp)$. Then $(\pounds^c,\wp)$ is a sSb^*-open. Let $\{(_\epsilon,\wp):\epsilon\in\ell\}$ be a sSb^*-open cover of $(\pounds,\wp)$. Therefore, $\{(_\epsilon,\wp):\epsilon\in\zeta\}\cup(\pounds^c,\wp)$ is sSb^*-open cover of $\widetilde{\mathcal{W}}$. For $\widetilde{\mathcal{W}}$ is sSb^*- compact space, $\exists$ finite subcover $\{(_\epsilon,\wp):\epsilon\in\zeta_0\}\cup(\pounds^c,\wp)$ for $\widetilde{\mathcal{W}}$. Now, $[\{(_\epsilon,\wp):\epsilon\in\zeta_0\}\cup(\pounds^c,\wp)]-(\pounds^c,\wp)$ is a finite subcover of $\{(_\epsilon,\wp):\epsilon\in\zeta\}$ for $(\pounds,\wp)$. So, $(\pounds,\wp)$ is sSb^*-compact.
□

Theorem 2.12. *Every sSb^*-closed subset $(\mathcal{S},\wp)$ of sSb^*-Lindelöf space $(\mathcal{W},\mathcal{Q},\wp)$ is sSb^*-Lindelöf.*

Proof. Assume that $(\mathcal{S},\wp)$ be sSb^*-closed subset $(\mathcal{S},\wp)$ of sSb^*-compact space $(\mathcal{W},\mathcal{Q},\wp)$ and $\{(\Psi_\epsilon,\wp):\epsilon\in\zeta\}$ be sSb^*-open cover of $(\mathcal{S},\wp)$. Therefore, $(\mathcal{S}^c,\wp)$ is a sSb^*-open and $(\mathcal{S}^c,\wp)\subseteq\cup_{\epsilon\in\zeta}(\Psi_\epsilon,\wp)$. Therefore, $\widetilde{\mathcal{W}}=(\Psi_\epsilon,\wp)\cup(\mathcal{S}^c,\wp)$. Since $\widetilde{\mathcal{W}}$ is a sSb^*-Lindelöf space, then $\widetilde{\mathcal{W}}=\cup_{\epsilon\in\zeta}(\Psi_\epsilon,\wp)\cup(\mathcal{S}^c,\wp)$. This implies that $(\mathcal{S},\wp)\subseteq\cup_{\epsilon\in\zeta}(\Psi_\epsilon,\wp)$. Hence, $(\mathcal{S},\wp)$ is a sSb^*-Lindelöf. □

Corollary 2.13. *If $(\delta,\wp)$ is a sSb^*-closed subset of $\widetilde{\mathcal{W}}$ and $(\Psi,\wp)$ is a sSb^*-compact (resp. sSb^*-Lindelöf) subset of $\widetilde{\mathcal{W}}$. Then, $(\Psi,\wp)\cap(\delta,\wp)$ is a sSb^*-compact (resp. sSb^*-Lindelöf).*

Proof. Let $(\Psi,\wp)$ be a sSb^*-compact set, consider $\{(G_\varepsilon,\wp):\varepsilon\in\zeta\}$ is sSb^*-open cover of $(\Psi,\wp)\cap(\delta,\wp)$. Then $\{(G_\varepsilon,\wp):\varepsilon\in\zeta\}\cup(\delta^c,\wp)$ is sSb^*-open cover of $(\Psi,\wp)$. For $(\Psi,\wp)$ is a sSb^*-compact. So, $\exists$ a soft finite subfamily ζ_0 of $\zeta\ni(\Psi,\wp)\subseteq\cup_{\varepsilon\in\zeta_0}(G_\varepsilon,\wp)\cup(\delta^c,\wp)$. Hence, $(\Psi,\wp)\cap(\delta,\wp)\subseteq\cup_{\varepsilon\in\zeta_0}(G_\varepsilon,\wp)\cap(\delta,\wp)\subseteq\cup_{\varepsilon\in\zeta_0}(G_\varepsilon,\wp)$. Therefore, $(\Psi,\wp)\cap(\delta,\wp)$ is sSb^*-compact.
The same evidence applies to sSb^*-Lindelöf space. □

In the next example, indicates that the inclusions of the Theorem 2.12 is not necessary correct.

Example 3. Let $\mathcal{W}=\{h_1,h_2\}$ and $\wp=\{\nabla_1,\nabla_2\}$. Consider
$\mathcal{Q}=\{\widetilde{\mathcal{W}},\widetilde{\phi},(\Psi_1,\wp),(\Psi_2,\wp),(\Psi_3,\wp)\}$ where $(\Psi_1,\wp),(\Psi_2,\wp)$ and $(\Psi_3,\wp)$ defined as following manner:
$(\Psi_1,\wp)=\{(\nabla_1,\{h_1\}),(\nabla_2,\phi)\}$,
$(\Psi_2,\wp)=\{(\nabla_1,\phi),(\nabla_2,\{h_2\})\}$ and
$(\Psi_3,\wp)=\{(\nabla_1,\{h_1\}),(\nabla_2,\{h_2\})\}$.
Then $(\mathcal{W},\mathcal{Q},\wp)$ is a $\mathcal{STS}$ over $\mathcal{W}$. Obviously, $(\mathcal{W},\mathcal{Q},\wp)$ is sSb^*-compact. furthermore, a soft set $(\Pi,\wp)=\{(\nabla_1,\{h_1\}),(\nabla_2,\mathcal{W})\}$ is a sSb^*-compact, even so it is not a sSb^*-closed.

Theorem 2.14. *A $(\mathcal{W},\mathcal{Q},\wp)$ is sSb^*-compact (resp. sSb^*-Lindelöf) if and only if each collection of sSb^*-closed subsets of $(\mathcal{W},\mathcal{Q},\wp)$, satisfying the soft finite (resp. soft countable) intersection property, $\cap_{\varepsilon\in\ell}(\Psi_\varepsilon,\wp)\neq\phi$.*

Proof. Let $(\mathcal{W},\mathcal{Q},\wp)$ is a sSb^*-compact, and $\{(\xi_\epsilon,\wp):\epsilon\in\ell\}$ be a family of sSb^*-closed subsets of $\widetilde{\mathcal{W}}$. Let $\cap_{\epsilon\in\ell}(\xi_\epsilon,\wp)=\phi$. Then $\widetilde{\mathcal{W}}=\cup_{\epsilon\in\ell}(\xi^c_\epsilon,\wp)$. For $\cup_{\epsilon\in\ell}(\xi^c_\epsilon,\wp)$ is a collection of sSb^*-open sets covering $\widetilde{\mathcal{W}}$. As $(\mathcal{W},\mathcal{Q},\wp)$ is sSb^*-compact, then $\exists$ a soft finite subset ℓ_0 of $\ell\ni\cup_{\epsilon\in\ell_0}(\xi^c_\epsilon,\wp)=\widetilde{\mathcal{W}}$ then $\cap_{\epsilon\in\ell_0}(\xi_\epsilon,\wp)=\phi$. Which gives contradictions. Therefore, $\cap_{\epsilon\in\ell}(\xi_\epsilon,\wp)\neq\phi$. Conversely, let $\{(\gamma_\epsilon,\wp):\epsilon\in\ell\}$ be a family of sSb^*-open cover of $\widetilde{\mathcal{W}}$. Let for every finite subset $\ell_0\subset\ell$, we have $\cup_{\epsilon\in\ell_0}(\gamma^c_\epsilon,\wp)\neq\widetilde{\mathcal{W}}$. Then $\cap_{\epsilon\in\ell}(\gamma^c_\epsilon,\wp)\neq\phi$. Thus, $\{(\gamma^c_\epsilon,\wp):\epsilon\in\ell\}$ satisfies the finite intersection property. By definition get $\cap_{\epsilon\in\ell}(\gamma^c_\epsilon,\wp)\neq\phi$ which implies $\cup_{\epsilon\in\ell_0}(\gamma_\epsilon,\wp)\neq\widetilde{\mathcal{W}}$ and this contradicts that $\{(\gamma_\epsilon,\wp):\epsilon\in\ell\}$ is a sSb^*-open cover of $\widetilde{\mathcal{W}}$.
Hence, $(\mathcal{W},\mathcal{Q},\wp)$ is a sSb^*-compact space. □

THEOREM 2.15. *Let $\Pi:\mathcal{W}\to\Sigma$ be a sSb^*-continuous function. If $\mathcal{W}$ is a sSb^*-compact space, then the image of $\mathcal{W}$ under the Π is a soft compact.*

Proof. Assume $\Pi:\mathcal{W}\to\Sigma$ is a sSb^*-continuous, $\{(G_\varepsilon,\wp):\varepsilon\in\ell\}$ is a soft cover of Σ. For Π is a sSb^*-continuous, therefore $\{\Pi^{-1}((G_\varepsilon,\wp)):\varepsilon\in\ell\}$ is a sSb^*-open cover of $\widetilde{\mathcal{W}}$ and $\mathcal{W}$ is a sSb^*-compact, $\exists$ a soft finite sub-set ℓ_0 of $\ell\ni\{\Pi^{-1}((G_\varepsilon,\wp)):\varepsilon\in\ell_0\}$ composes a sSb^*-open cover of $\widetilde{\mathcal{W}}$. Thus, $\{\Pi^{-1}((G_\varepsilon,\wp)):\varepsilon\in\ell_0\}$ composes a soft finite soft open cover of $\widetilde{\Sigma}$. Therefore, Σ is a soft compact. □

THEOREM 2.16. *Let $\Pi:\mathcal{W}\to\Sigma$ be a sSb^*-irresolute surjection and $\mathcal{W}$ is a sSb^*-compact space, then Σ is a sSb^*-compact.*

Proof. Suppose $\Pi:\mathcal{W}\to\Sigma$ be a sSb^*-irresolute surjection, $\mathcal{W}$ be a sSb^*-compact $(\mathcal{W},\mathcal{Q},\wp)$ to (Σ,Ω,Θ). A soft open cover $\{(\delta_\epsilon,\wp):\epsilon\in\zeta\}$ of Σ. Then $\{\Pi^{-1}((\delta_\epsilon,\wp)):\epsilon\in\zeta\}$ is a sSb^*-open cover of $\widetilde{\mathcal{W}}$. For $\mathcal{W}$ is a sSb^*-compact, then $\exists$ a finite subset ζ_0 of ζ such that $\{\Pi^{-1}((\delta_\epsilon,\wp)):\epsilon\in\zeta_0\}$ composes a sSb^*-open cover of $\widetilde{\mathcal{W}}$. Therefore, $\{\Pi^{-1}((\delta_\epsilon,\wp)):\epsilon\in\zeta_0\}$ composes a finite sSb^*-open cover of $\widetilde{\Sigma}$. Hence, Σ is a sSb^*-compact. □

THEOREM 2.17. *The sSb^*-irresolute image of a sSb^*-compact (resp. sSb^*-Lindelöf) set is a sSb^*-compact (resp. sSb^*-Lindelöf).*

Proof. Assume that $\Pi:\mathcal{W}\to\Sigma$ be a sSb^*-irresolute and let $(,\wp)$ be a sSb^*-Lindelöf subset of $\widetilde{\mathcal{W}}$. Let $\{(\Psi_\epsilon,\wp):\epsilon\in\zeta\}$ is sSb^*-open cover of $\Pi(,\wp)$. Then $\Pi(,\wp)\subseteq\cup_{\epsilon\in\zeta}(\Psi_\epsilon,\wp)$. Then, $(,\wp)\subseteq\cup_{\epsilon\in\zeta}\Pi^{-1}(\Psi_\epsilon,\wp)$ and $\Pi^{-1}(\Psi_\epsilon,\wp)$ is sSb^*-open, for every $\epsilon\in\zeta$. by assumption, $(,\wp)$ is a sSb^*-Lindelöf, then $(,\wp)\subseteq\cup_{\epsilon\in\zeta}\Pi^{-1}(\Psi_\epsilon,\wp)$. Therefore, $\Pi(,\wp)\subseteq\cup_{\epsilon\in\zeta}\Pi(\Pi^{-1}(\Psi_\epsilon,\wp))\subseteq\cup_{\epsilon\in\zeta}(\Psi_\epsilon,\wp)$. Thus, $\Pi(,\wp)$ is sSb^*-Lindelöf space.
The same proof in case sSb^*-compact space. □

3. Soft strongly b^*-connected spaces. One of the most important properties of soft strongly b^*-connected space is discussed and explored in this section.

DEFINITION 3.1. *Let $(\mathcal{W},\mathcal{Q},\wp)$ be $\mathcal{STS}$, and $(\Psi,\wp),(\pounds,\wp)$ are sSb^*-open sets over $\widetilde{\mathcal{W}}$. Then, $(\Psi,\wp)$ and $(\pounds,\wp)$ are stated to be soft strongly b^*-separated sets iff $sSb^*cl(\Psi,\wp)\cap(\pounds,\wp)=\phi$ and $(\Psi,\wp)\cap sSb^*cl(\pounds,\wp)=\phi$.*

THEOREM 3.2. *If $(\Psi,\wp)$ and $(\pounds,\wp)$ are sSb^*-separated sets then they are disjoint.*

Proof. $(\Psi,\wp)\cap(\pounds,\wp)\subseteq sSb^*cl(\Psi,\wp)\cap(\pounds,\wp)=\phi$. □

THEOREM 3.3. *If $(\Psi,\wp)$ and $(\pounds,\wp)$ are sSb^*-separated subsets of $\mathcal{W}$ and $(\Gamma,\wp)\subseteq(\Psi,\wp)$ and $(\Upsilon,\wp)\subseteq(\pounds,\wp)$ then $(\Gamma,\wp)$ and $(\Upsilon,\wp)$ are also sSb^*-separated.*

Proof. Suppose $(\Psi,\wp)$ and $(\pounds,\wp)$ are sSb^*-separated subsets of a space $\mathcal{W}$, by definition 3.1; $sSb^*cl(\Psi,\wp)\cap(\pounds,\wp)=\phi$ and $(\Psi,\wp)\cap sSb^*cl(\pounds,\wp)=\phi$. Since $(\Gamma,\wp)\subseteq(\Psi,\wp)$, we have $sSb^*cl(\Gamma,\wp)\subseteq sSb^*cl(\Psi,\wp)$ and since $(\Upsilon,\wp)\subseteq(\pounds,\wp)$, then $sSb^*cl(\Upsilon,\wp)\subseteq sSb^*cl(\pounds,\wp)$. Hence, $(\Gamma,\wp)\cap sSb^*cl(\Upsilon,\wp)=(\Psi,\wp)\cap sSb^*cl(\pounds,\wp)=\phi$ and $sSb^*cl(\Gamma,\wp)\cap(\Upsilon,\wp)=sSb^*cl(\Psi,\wp)\cap(\pounds,\wp)=\phi$. Therefore, $(\Gamma,\wp)$ and $(\Upsilon,\wp)$ are also sSb^*-separated. □

THEOREM 3.4. *Two soft separated sets are soft sSb^*-separated sets.*

Proof. Assume $(\vartheta,\wp)$ and $(,\wp)$ be two soft separated sets over $\mathcal{W}$, so $sSb^*cl(\vartheta,\wp)\cap(,\wp)=\phi$ and $(\vartheta,\wp)\cap sSb^*cl(,\wp)=\phi$.
As
$sSb^*cl(\vartheta,\wp)\subseteq cl(\vartheta,\wp)$.
$sSb^*cl(\vartheta,\wp)\cap(,\wp)\subseteq cl(\vartheta,\wp)\cap(,\wp)=\phi$.
and similarly, $(\vartheta,\wp)\cap sSb^*cl(,\wp)\subseteq(\vartheta,\wp)\cap cl(,\wp)=\phi$.
Hence, $(\vartheta,\wp)$ and $(,\wp)$ are sSb^*-separated sets.
□

Remark 1. If $(\vartheta,\wp)$ and $(,\wp)$ are disjoint. Then, require not be sSb^*-separated.

Example 4. Consider $\mathcal{W} = \{\varsigma_1, \varsigma_2\}$ and $\wp = \{\nabla_1, \nabla_2\}$. Consider $\mathcal{Q} = \{\widetilde{\mathcal{W}}, \widetilde{\tilde{\phi}}, (\vartheta, \wp)\}$ where $(\vartheta, \wp) = \{(\nabla_1, \{\varsigma_1\}), (\nabla_2, \{\varsigma_2\})\}$,
let $(, \wp) = \{(\nabla_1, \{\varsigma_1\}), (\nabla_2, \{\varsigma_2\})\}$ and $(\mathcal{S}, \wp) = \{(\nabla_1, \{\varsigma_2\}), (\nabla_2, \{\varsigma_1\})\}$ be two soft sets over $\mathcal{Q}$. Then $(, \wp)$ and $(\mathcal{S}, \wp)$ are soft disjoint sets but they are not sSb^*-separated.

Definition 3.5. *Let $(\mathcal{W}, \mathcal{Q}, \wp)$ be $\mathcal{STS}$ over $\mathcal{W}$. Then $(\mathcal{W}, \mathcal{Q}, \wp)$ is stated to be sSb^*-connected, if $\mathcal{W}$ cannot be intimated as the union of two sSb^*-open sets. Else, $(\mathcal{W}, \mathcal{Q}, \wp)$ is stated to be a sSb^*-disconnected.*

Example 5. Consider $\mathcal{W} = \{r, t, d\}$ and $\wp = \{\nabla_1, \nabla_2\}$. Consider
$\mathcal{Q} = \{\widetilde{\mathcal{W}}, \widetilde{\tilde{\phi}}, (\Gamma_1, \wp), (\Gamma_2, \wp), (\Gamma_3, \wp), (\Gamma_4, \wp), (\Gamma_5, \wp)\}$
where $(\Gamma_1, \wp), (\Gamma_2, \wp), (\Gamma_3, \wp), (\Gamma_4, \wp)$ and $(\Gamma_5, \wp)$ are sSb^*-open sets over $\mathcal{W}$, define as follows:
$(\Gamma_1, \wp) = \{(\nabla_1, \{t\}), (\nabla_2, \{r\})\}$,
$(\Gamma_2, \wp) = \{(\nabla_1, \{t, d\}), (\nabla_2, \{r, t\})\}$,
$(\Gamma_3, \wp) = \{(\nabla_1, \{r, t\}), (\nabla_2, \mathcal{W})\}$,
$(\Gamma_4, \wp) = \{(\nabla_1, \{r, t\}), (\nabla_2, \{r, d\})\}$ and
$(\Gamma_5, \wp) = \{(\nabla_1, \{t\}), (\nabla_2, \{r, t\})\}$.
Then $(\mathcal{W}, \mathcal{Q}, \wp)$ is $\mathcal{STS}$ on $\mathcal{W}$. Thus, $(\mathcal{W}, \mathcal{Q}, \wp)$ is a $\mathcal{STS}$ over $\mathcal{W}$. Intelligibly, $\mathcal{W}$ is a sSb^*-connected.

Theorem 3.6. *Let $(\mathcal{W}, \mathcal{Q}, \wp)$ be a $\mathcal{STS}$ and $(\Psi, \wp)$ is a sSb^*-connected. Let $(\pounds, \wp)$ and $(\mathcal{S}, \wp)$ are sSb^*-separated sets. If $(\Psi, \wp) \subseteq (\pounds, \wp) \cup (\mathcal{S}, \wp)$. Then either $(\Psi, \wp) \subseteq (\pounds, \wp)$ or $(\Psi, \wp) \subseteq (\mathcal{S}, \wp)$.*

Proof. Suppose $(\Psi, \wp)$ be a sSb^*-connected and $(\pounds, \wp)$, $(\mathcal{S}, \wp)$ are sSb^*-separated sets such that $(\Psi, \wp) \subseteq (\pounds, \wp) \cup (\mathcal{S}, \wp)$. Let $(\Psi, \wp)$ not subset of $(\pounds, \wp)$ and $(\Psi, \wp)$ not subset of $(\mathcal{S}, \wp)$. Suppose $(P_1, \wp) \subseteq (\pounds, \wp) \cap (\Psi, \wp) \neq \phi$ and $(P_2, \wp) \subseteq (\mathcal{S}, \wp) \cap (\Psi, \wp) \neq \phi$. Then $(\Psi, \wp) = (P_1, \wp) \cup (P_2, \wp)$. Since $(P_1, \wp) \subseteq (\pounds, \wp)$, hence $sSb^*cl(P_1, \wp) \subseteq sSb^*cl(\pounds, \wp)$. Since $sSb^*cl(\pounds, \wp) \cap (\mathcal{S}, \wp) = \phi$ then $sSb^*cl(P_1, \wp) \cap (P_2, \wp) = \phi$. Since $(P_2, \wp) \subseteq (\mathcal{S}, \wp)$, hence $sSb^*cl(P_2, \wp) \subseteq sSb^*cl(\mathcal{S}, \wp)$. Since $sSb^*cl(\mathcal{S}, \wp) \cap (\pounds, \wp) = \phi$, then $sSb^*cl(P_2, \wp) \cap (P_1, \wp) = \phi$. But $(\Psi, \wp) = (P_1, \wp) \cup (P_2, \wp)$. Therefore, $(\Psi, \wp)$ is not a sSb^*-connected. This is a contradiction. Then either $(\Psi, \wp) \subseteq (\pounds, \wp)$ or $(\Psi, \wp) \subseteq (\mathcal{S}, \wp)$. □

Theorem 3.7. *Let $(\Psi, \wp)$ be a sSb^*-connected set. If $(\Psi, \wp) \subseteq (\pounds, \wp) \subseteq sSb^*cl(\Psi, \wp)$ then $(\pounds, \wp)$ is also a sSb^*-connected.*

Proof. If $(\Psi, \wp)$ be not a sSb^*-connected, then $\exists$ two soft sets $(\mathcal{S}, \wp) \subseteq (G, \wp)$ such that $sSb^*cl(\mathcal{S}, \wp) \cap (G, \wp) = (G, \wp) \cap sSb^*cl(\mathcal{S}, \wp) = \phi$ and $(\pounds, \wp) = (\mathcal{S}, \wp) \cup (G, \wp)$. Since $(\Psi, \wp) \subseteq (\pounds, \wp)$, thus either $(\Psi, \wp) \subseteq (\mathcal{S}, \wp)$ or $(\Psi, \wp) \subseteq (G, \wp)$. Suppose $(\Psi, \wp) \subseteq (\mathcal{S}, \wp)$ then $sSb^*cl(\Psi, \wp) \subseteq sSb^*cl(\mathcal{S}, \wp)$, thus $sSb^*cl(\Psi, \wp) \subseteq (G, \wp) = sSb^*cl(\mathcal{S}, \wp) \cap (G, \wp) = \phi$. But $(G, \wp) \subseteq (\pounds, \wp) \subseteq sSb^*cl(\Psi, \wp)$ thus $sSb^*cl(\Psi, \wp) \cap (G, \wp) = (G, \wp)$. Therefore, $(G, \wp) = \phi$, so is a contradiction. Hence, $(\pounds, \wp)$ is a sSb^*-connected. Similarly, if $(\Psi, \wp) \subseteq (\pounds, \wp)$, then $(\mathcal{S}, \wp) = \phi$. Which again a contradiction. Hence, $(\pounds, \wp)$ is a sSb^*-connected. □

Theorem 3.8. *If $(\Psi, \wp)$ is a sSb^*-connected set then $sSb^*cl(\Psi, \wp)$ is a sSb^*-connected.*

Proof. Let $(\Psi, \wp)$ is a sSb^*-connected set then $sSb^*cl(\Psi, \wp)$ is not. Then there exists two sSb^*-separation sets $(\mathcal{S}, \wp)$ and $(\delta, \wp)$ such that $sSb^*cl(\Psi, \wp) = (\mathcal{S}, \wp) \cup (\delta, \wp)$. But $(\Psi, \wp) \subseteq sSb^*cl(\Psi, \wp)$, then $(\Psi, \wp) = (\mathcal{S}, \wp) \cup (\delta, \wp)$ and since $(\Psi, \wp)$ is sSb^*-connected set, then by theorem 3.6 either $(\Psi, \wp) \subseteq (\mathcal{S}, \wp)$ or $(\Psi, \wp) \subseteq (G, \wp)$. If $(\Psi, \wp) \subseteq (\mathcal{S}, \wp)$ then $sSb^*cl(\Psi, \wp) \subseteq sSb^*cl(\mathcal{S}, \wp)$. But $sSb^*cl(\mathcal{S}, \wp) \cap (\delta, \wp) = \phi$. Hence, $sSb^*cl(\Psi, \wp) \cap (\delta, \wp) = \phi$. Since $(\delta, \wp) \subseteq sSb^*cl(\Psi, \wp)$, then $(\delta, \wp) = \phi$. So is a contradiction. If $(\Psi, \wp) \subseteq (\mathcal{S}, \wp)$ we can prove $(\mathcal{S}, \wp) = \phi$ as the same, that is a contradiction. Hence, $sSb^*cl(\Psi, \wp)$ is a sSb^*-connected. □

Theorem 3.9. *If $(\Psi, \wp)$ and $(\mathcal{S}, \wp)$ are two sSb^*-connected sets where $(\Psi, \wp) \cap (\mathcal{S}, \wp) \neq \phi$. Therefore $(\Psi, \wp) \cup (\mathcal{S}, \wp)$ is also a sSb^*-connected set.*

Proof. Assume that, if possible, $(\Psi, \wp) \cup (\mathcal{S}, \wp)$ be sSb^*-disconnected set, then $(\Psi, \wp) \cup (\mathcal{S}, \wp) = (\vartheta, \wp) \cup (\delta, \wp)$, where $(\vartheta, \wp) \neq \phi, (\delta, \wp) \neq \phi \ni (\vartheta, \wp)$ and $(\delta, \wp)$ are sSb^*-separation. Since $(\Psi, \wp) \subseteq (\Psi, \wp) \cup (\mathcal{S}, \wp) = (\vartheta, \wp) \cup (\delta, \wp)$, Therefore, $(\Psi, \wp) \subseteq (\vartheta, \wp) \cup (\delta, \wp)$. Hence, by Theorem 3.6, we have either $(\Psi, \wp) \subseteq (\vartheta, \wp)$ or $(\Psi, \wp) \subseteq (\delta, \wp)$. Again, either $(\mathcal{S}, \wp) \subseteq (\vartheta, \wp)$ or $(\mathcal{S}, \wp) \subseteq (\delta, \wp)$. Thus, we have four choices either $(\Psi, \wp) \subseteq (\vartheta, \wp)$ and $(\mathcal{S}, \wp) \subseteq (\vartheta, \wp)$ or $(\Psi, \wp) \subseteq (\vartheta, \wp)$ and $(\mathcal{S}, \wp) \subseteq (\delta, \wp)$ or $(\Psi, \wp) \subseteq (\delta, \wp)$ and $(\mathcal{S}, \wp) \subseteq (\vartheta, \wp)$ or $(P, \wp \subseteq (\delta, \wp)$ and $(\mathcal{S}, \wp) \subseteq (\delta, \wp)$. If $(\Psi, \wp) \subseteq (\vartheta, \wp)$ and $(S, \wp \subseteq (\vartheta, \wp)$ or $(\Psi, \wp) \subseteq (\delta, \wp)$ and $(\mathcal{S}, \wp) \subseteq (\delta, \wp)$, then $(\Psi, \wp) \cup (\mathcal{S}, \wp) \subseteq (\vartheta, \wp)$ or $(\Psi, \wp) \cup (\mathcal{S}, \wp) \subseteq (\delta, \wp) \Rightarrow (\vartheta, \wp) \cup (\delta, \wp) \subseteq (\vartheta, \wp)$ or $(\vartheta, \wp) \cup (\delta, \wp) \subseteq (\delta, \wp) \Rightarrow (\vartheta, \wp) \cup (\delta, \wp) = (\vartheta, \wp)$ or $(\vartheta, \wp) \cup (\delta, \wp) = (\delta, \wp) \Rightarrow (\delta, \wp) = \phi$ or $(\vartheta, \wp) = \phi$, then is a contradiction. So $(\Psi, \wp) \subseteq (\vartheta, \wp)$ and $(\mathcal{S}, \wp) \subseteq (\delta, \wp)$ or $(\Psi, \wp) \subseteq (\delta, \wp)$ and $(\mathcal{S}, \wp) \subseteq (\vartheta, \wp)$, then in both the cases, $(\Psi, \wp) \cap (\mathcal{S}, \wp) \subseteq (\vartheta, \wp) \cap (\delta, \wp) = \phi \Rightarrow (\Psi, \wp) \cap (\mathcal{S}, \wp) = \phi$. So, is contradiction again to the given supposition that $(\Psi, \wp) \cap (\mathcal{S}, \wp) \neq \phi$. Hence, we have $(\Psi, \wp) \cup (\mathcal{S}, \wp)$ is also a sSb^*-connected set. □

THEOREM 3.10. *If $(\mathcal{W}, \mathcal{Q}, \wp)$ is a sSb^*-connected space, then it is a soft connected.*

Proof. Suppose $(\mathcal{W}, \mathcal{Q}, \wp)$ not sSb^*-disconnected. Therefore, $\exists$ nonnull soft sets $(\pounds, \wp)$ and $(\mathcal{S}, \wp)$, where $\mathcal{W} = (\pounds, \wp) \cup (\mathcal{S}, \wp) \ni cl(\pounds, \wp) \cap (\mathcal{S}, \wp) = \phi$ and $(\pounds, \wp) \cap cl(\mathcal{S}, \wp) = \phi$. Since $sSb^*cl(\pounds, \wp) \subseteq cl(\pounds, \wp)$. Thus, $sSb^*cl(\pounds, \wp) \cap (\mathcal{S}, \wp) \subseteq cl(\pounds, \wp) \cap (\mathcal{S}, \wp) = \phi$. Hence, $sSb^*cl(\pounds, \wp) \cap (\mathcal{S}, \wp) = \phi$. Similarly, $(\pounds, \wp) \cap sSb^*cl(\mathcal{S}, \wp) = \phi$. Hence, $(\mathcal{W}, \mathcal{Q}, \wp)$ is a sSb^*-disconnected. So, is a contradiction. Therefore, $(\mathcal{W}, \mathcal{Q}, \wp)$ is a soft connected. □

THEOREM 3.11. *If $\Omega : \mathcal{W} \to \Sigma$ be a sSb^*-continuous surjection and $\mathcal{W}$ is a sSb^*-connected space, then Σ is soft connected.*

Proof. Assume that Σ is not soft connected. Let $\Sigma = (\Psi, \wp) \cup (\pounds, \wp)$ where $(\Psi, \wp)$ and $(\pounds, \wp)$ are disjoint nonempty soft open sets in Σ. Since Ω is a sSb^*-continuous and onto, $\mathcal{W} = \Omega^{-1}(\Psi, \wp) \cup \Omega^{-1}(\pounds, \wp)$ where $\Omega^{-1}(\Psi, \wp)$ and $\Omega^{-1}(\pounds, \wp)$ are disjoint nonempty sSb^*-open sets in $\mathcal{W}$, which is contradiction to $\mathcal{W}$ is a sSb^*-connected. Therefore, Σ is a soft connected. □

THEOREM 3.12. *If $\Omega : \mathcal{W} \to \Sigma$ is a sSb^*-irresolute surjection and $\mathcal{W}$ is a sSb^*-connected, then Σ is a sSb^*-connected.*

Proof. Assume Σ is not sSb^*-connected and $\Sigma = (\Psi, \wp) \cup (\pounds, \wp)$ where $(\Psi, \wp)$ and $(\pounds, \wp)$ are disjoint nonempty sSb^*-open sets in Σ. Since Ω is a sSb^*-irresolute and onto, $\mathcal{W} = \Omega^{-1}(\Psi, \wp) \cup \Omega^{-1}(\pounds, \wp)$ where $\Omega^{-1}(\Psi, \wp)$ and $\Omega^{-1}(\pounds, \wp)$ are disjoint nonempty sSb^*-open sets in $\mathcal{W}$, which is contradiction to $\mathcal{W}$ is a sSb^*-connected. Therefore, Σ is a sSb^*-connected. □

4. Conclusion. In this article, we presented some of conception of soft sets and soft topological spaces are investigated. The basis of paper is to establish and introduce soft compactness and soft Lindelöfness, namely, sSb^*-compactness, sSb^*-Lindelöfness. Examining some properties of these spaces allows us to prove some of our results and varied introduce the relationship between spaces and illustrate our main findings. Moreover, We define and explore the soft strongly b^*-connected spaces and discuss its relation with soft connectedness spaces. Also, the properties of sSb^*-connected and sSb^*-disconnected with examples are studied.

5. Acknowledgements. The authors would like to thank Mustansiriyah University, (https://uomustansiriyah.edu.iq/), Baghdad, Iraq, for its support to the present work. Also, the authors would like to thank the Reviewers for their valuable comments which help us to improve the manuscript.

REFERENCES

[1] D. Molodtsov, *Soft set theory-first results*, Computers and Mathematics with Applications, vol. 37, no. 4-5, pp. 19-31, 1999.
[2] Shabir, M., Naz, M., *On soft topological spaces*, Comput. Math. Appl. 61, pp. 1786–1799, 2011.
[3] K. Kannan, *Soft generalized closed sets in soft topological spaces*, journal of Theoretical and Appl. Inform. Technology, 37 (1) pp.17 – 21, 2012.
[4] Metin Akdag, Alkan Ozkan, *Soft $\alpha-$open sets and soft $\alpha-$continuous functions*, Abstr. Anal. Appl. Art ID 891341, pp. 1-7, 2014.
[5] Metin Akdag, Alkan Ozkan, *Soft $b-$open and soft $b-$continuous functions*, Math Sci 8:124, 2014.
[6] I. Zorlutuna, M. Akdag, W. K. Min, and S. Atmaca, *Remarks on soft topological spaces*, Annals of Fuzzy Mathematics and Informatics, vol. 3, no. 2, pp. 171-185, 2012.
[7] J. R. Porter and R. G. Woods, *Subspaces of connected spaces*, Topology and Its Applications, vol. 68, pp. 113 – 131, 1996. DOI: 10.1016/0166-8641(95)00057-7.
[8] S. Hussain, *A note on soft connectedness*, journal of the Egyptian Mathematical Society 23, pp. 6 – 11, 2015. DOI: 10.1016/j.joems.2014.02.003.
[9] Hameed, S. Z. and Hussein, A. K., *On soft bc$-$open sets in soft topological spaces*, Iraqi Journal of Science, pp. 238-242. 2020.
[10] Hameed, Saif Z., Fayza A. Ibrahem, and Essam A. El-Seidy, *On soft b^*-closed sets in soft topological space* International journal of Nonlinear Analysis and Applications, 12.1 pp. 1235-1242, 2021. https://doi.org/10.24996/ijs.2020.SI.1.32.
[11] Hameed, Saif Z., Abdelaziz E. Radwan, and Essam A. El-Seidy, *On soft b^*-Continuous functions in soft topological space* Measurement: Sensors, vol. 27, pp. 1-5, 2023.
[12] Hameed, Saif Z., Abdelaziz E. Radwan, and Essam A. El-Seidy, *On soft strongly b^*-closed sets and soft strongly b^*-continuous functions in soft topological space* Under the Publication, 2023.
[13] Maji, P.K., Biswas, R., Roy, A.R., *Soft set theory*, Comput. Math. Appl. 45, pp. 555–562, 2003.
[14] I. Arockiarani and A. Arokialancy, *Generalized soft $g\beta-$closed sets and soft $gs\beta-$closed sets in soft topological spaces* International journal of Mathemat- ical Archive, vol. 4, no. 2, pp. 1-7, 2013.
[15] B. Chen, *Soft semi-open sets and related properties in soft topological spaces*, Applied Mathematics and Information Sciences, vol. 7, no. 1, pp. 287-294, 2013.

[16] Nirmala Rebecca Paul, *Remarks on soft $\omega-$closed sets in soft topological spaces*, Boletim da Sociedade Paranaense de Matemática, Vol. 33, No. 1, pp. 183-192, 2015. https://doi.org/10.5269/bspm.v33i1.22719.

Edited by: Mustafa M Matalgah
Special issue on: Synergies of Neural Networks, Neurorobotics, and Brain-Computer Interface Technology: Advancements and Applications
Received: Jan 13, 2024
Accepted: Mar 12, 2024