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INVESTIGATION ON THE OPTIMAL PROPERTIES OF SEMI ACTIVE CONTROL DEVICES WITH CONTINUOUS CONTROL FOR EQUIPMENT ISOLATION

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Abstract. The paper treats the semi active isolation of a single equipment, acceleration sensitive, by means of a variable elastic control device. A numerical study on a single degree of freedom (SDOF) structural model equipped with a continuously variable elastic device subjected to harmonic input is presented. The utilized control algorithm is derived by the Lyapunov method and specialized in order to obtain instantaneous optimality. In order to minimize the dynamic response of interest, i.e. the equipment absolute acceleration, some parameters that define the algorithm and the device are conveniently selected. The purpose of the paper is to investigate the optimal isolation properties of semi active variable stiffness devices with continuous control across the whole frequency spectrum. The performances of the isolated equipment are evaluated in terms of absolute acceleration transmissibility. Semi active continuous control is compared with semi active ON-OFF mode and conventional linear passive control. Results show that it is possible to choose conveniently the parameters regulating semi active continuous control in order to limit the absolute acceleration transmissibility at all the frequencies. In literature from problems concerning vibration isolation, transmissibility is alternatively defined in terms of absolute acceleration or displacement. Here, absolute displacement transmissibility is also estimated. It is observed that in case of semi active control, there are differences between the two transmissibility representations, and they do not lead to analogous results for evaluating the performance of the control system.

Key words: Equipment isolation, semi active control, continuous control law, absolute acceleration and displacement transmissibility

AMS subject classifications. 34H35, 98C83

1. Introduction. Among the different control strategies, semi active devices appear interesting for vibration mitigation, since they can adjust adaptively their mechanical characteristics (stiffness and/or dissipation parameters) in real time depending on the input and/or the structural response according to a given control law, without adding external energy to the structures, thus not compromising system stability. Numerical and experimental works that focus the attention on the performances of semi active control for the dynamic response of systems with different structural configurations are present in literature, e.g. [1], [2]. The force supplied by semi active devices can vary according to a given control law which can be ON-OFF or continuous. In ON-OFF operation the control devices may assume only one of the two operation states: ON state (device activated) and OFF state (device deactivated). In continuous operation, the mechanical parameters of the semi active devices may continuously assume any value between the given limits. Most of the studies and applications found in the literature concern applications with ON-OFF control, e.g. [1]- [5], whereas only few papers concern continuous semi active control, [6]- [9], for this reason, closer attention will be given to this area in the following investigation.

Equipment of various nature, intended as objects of artistic great value (statues or sculptures), or precision technical equipment, or sensitive and refined medical components in the hospitals, placed in buildings usually need to be protected against vibrations. Among this class of objects, the study is concerned with acceleration sensitive components prone to damage from inertial loading. Passive systems have been proved to be effective and practical, but at the same time might suffer from low-frequency resonances and excessive isolator displacements, if subjected to long period ground motions (e.g. near-fault earthquakes). For this reason, efforts on the utilization of semi active devices for vibration isolation are made; the response transmitted to the equipment can be effectively reduced, and performances appear superior when compared with conventional linear passive control strategy [10]- [12].

This study investigates the base isolation of acceleration sensitive equipment by means of a variable elastic device with continuous control law. A single degree of freedom model is adopted with a harmonic base motion as input motion condition and the attention is focused on the absolute approach (absolute motion with respect to a fixed reference). The continuous control law is derived from the Lyapunov method and specialized in order to obtain instantaneous optimality. The aim of the paper is to investigate the optimal properties of semi active

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continuous control varying the frequency ratio between the input and the equipment, in order to cover all the regions of the frequency spectrum. Two parameters, one related to the algorithm and the other related to the device, defined as the stiffness ratio between the device and the equipment, are optimized in order to minimize the equipment acceleration. The performance of the optimized control system is evaluated in terms of absolute acceleration transmissibility curves in order to investigate the behavior across the whole frequency spectrum. Comparisons with semi active ON-OFF and conventional linear passive control are also carried out.

Since in literature from problems concerning vibration isolation, transmissibility is alternatively defined in terms of absolute acceleration or displacement (often interpreted similar), in this paper, absolute displacement transmissibility is estimated as well. The aim of the paper is also to investigate if there are differences between the two transmissibility representations in case of semi active control, and to see if they lead to analogous results for evaluating the performance of the control system.

The paper is organized as follows: section two reports the description of the single degree of freedom model together with the governing equations, section three presents the semi active control law, section four discusses the case study and, finally, section five carries out the results in terms of control system optimization and the performances of semi active continuous control compared with semi active ON-OFF and conventional linear passive control.

2. Description of the model and governing equations. The reference model is represented by a single degree of freedom (SDOF) undamped structural model, base-excited, Fig. 2.1.

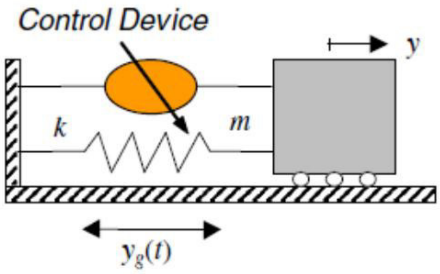

FIG. 2.1. *SDOF reference model.*

The state-space equations of motion, in the relative or absolute state space, can be written as:

$$\begin{aligned} \dot{\mathbf{z}}_{r,a}(t) &= \mathbf{A}\mathbf{z}_{r,a}(t) + \mathbf{B}u(t) + \mathbf{H}_{r,a}w_{r,a}(t) \\ \mathbf{z}_{r,a}(0) &= \mathbf{z}_{(r,a)0} \end{aligned} \tag{2.1}$$

where

$$\begin{aligned} \mathbf{z}_{r,a} = \begin{pmatrix} y \\ \dot{y}_{r,a} \end{pmatrix}; \mathbf{A} = \begin{pmatrix} 0 & 1 \\ -\omega^2 & 0 \end{pmatrix}; \mathbf{B} = \begin{pmatrix} 0 \\ -1/m \end{pmatrix} \\ \mathbf{H}_r = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \mathbf{H}_a = \begin{pmatrix} -1 \\ 0 \end{pmatrix}; w_r(t) = \ddot{y}_g(t); w_a(t) = \dot{y}_g(t) \end{aligned} \tag{2.2}$$

where ω is the circular frequency of the equipment, y is the relative displacement of the structural mass m with respect to support, y_g is the displacement of the support with respect to a fixed reference and u is the force in the control device. Finally, k is the elastic structural stiffness (k=$m\omega^2$) and T_0 is the natural period (T_0=$2\pi/\omega$). Absolute response quantities are evaluated by adding the support motion quantities to the corresponding relative ones. The present study refers to a variable stiffness device with control force:

$$u(t) = \lambda(t)m\omega^2[(y(t) - y(t_i)] \tag{2.3}$$

where the relative stiffness $\lambda(t)$ represents the ratio between the device (k_d) and the structural (k) stiffness, whereas $y(t_i)$ is the structural displacement corresponding to the last activation instant t_i of the device. The $\lambda(t)$ parameter has the following physical limitation:

$$0 \leq \lambda(t) \leq \lambda_{max} \tag{2.4}$$

3. Semi active control law. The direct Lyapounov method is used to define the control algorithm. It is based on the definition of an appropriate Lyapounov function and a control law which guarantees the hypothesis of the Lyapounov theorem. The state function, assumed as Lyapounov function, in the relative or absolute approach is defined as:

$$\Lambda = \frac{1}{2}\mathbf{z}_{r,a}^T(t)\mathbf{Q}\mathbf{z}_{r,a}(t) + \int_0^\tau \mathbf{u}^T\mathbf{R}\mathbf{u}(\tau)d\tau \tag{3.1}$$

where $\mathbf{Q}$ is the weighting matrix (symmetrical and, at least, positive semi-definite) of the state and $\mathbf{R}$ (symmetrical and positive definite) weights the control force. The procedure in order to select an optimal control law based on the Lyapounov function defined in Eq. 3.1 is deeply explained in [6] in the case of the relative approach. In this work only the final expression of the optimal control force is reported, valid for the relative and the absolute approach:

$$\mathbf{u}_{opt}(t) = -\frac{1}{2}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{Q}\mathbf{z}_{r,a}(t) \tag{3.2}$$

It is important to remark that the resulting law Eq. 3.2 guarantees only locally (for a given time) the optimality of the control law (i.e. a local minimum of the state function). It is possible to give a physical interpretation of the control process since the Lyapunov function Eq. 3.1 may be also viewed as input energy. In the case of a SDOF system, matrices appearing in Eq. 3.2 may be written as follows:

$$Q = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{pmatrix}; \mathbf{R} = R \tag{3.3}$$

The non-dimensional parameters and the optimized parameters for the algorithm introduced in [6] are used and, after passages, the optimal control force is:

$$u_{opt}(t) = \frac{m\omega}{\rho}\dot{y}_{r,a}(t) \tag{3.4}$$

For a variable stiffness semi active device with constitutive equation expressed by Eq. 2.3, the variation law of the device stiffness is obtained as follows:

$$\lambda(t) = F[0, \lambda^*(t), \lambda_{max}] \tag{3.5}$$

where for the relative approach the $\lambda^*(t)$ parameter is evaluated as:

$$\lambda^*(t) = \frac{1}{\rho\omega}\frac{\dot{y}_r(t)}{[(y(t) - y(t_i)]} \tag{3.6}$$

and for the absolute approach is:

$$\lambda^*(t) = \frac{1}{\rho\omega}\frac{\dot{y}_a(t)}{[(y(t) - y(t_i)]} \tag{3.7}$$

Let's observe that for $\rho \longrightarrow 0$, $u_{opt} \longrightarrow \infty$ and, by evaluating the sign of this limit, the ON-OFF control law is obtained, [1]. In both approaches, the parameter ρ must be selected.

4. Description of the case study. The study of the forced vibrations of the SDOF system represented in Fig. 2.1 is carried out, having assumed a harmonic base motion. In the Relative Approach (RA), the input is defined in terms of a base acceleration:

$$\ddot{y}_g(t) = A_g \sin(\beta\omega t + \psi) \tag{4.1}$$

where $\beta = \omega_f/\omega$ is the ratio between the input ω_f and the equipment ω circular frequency, A_g is the amplitude and ψ is the phase angle. In this work it is assumed A_g =1, since the structural system is homogeneous of order one [12] and the dynamic response is not dependent of the motion amplitude, and ψ =0.

In the Absolute Approach (AA) the input is defined in terms of a base velocity:

$$\dot{y}_g(t) = V_g \cos(\beta\omega t) \tag{4.2}$$

where $V_g = -A_g/\beta\omega$.

Concerning the control system, an optimal selection of the algorithm parameter ρ and the relative stiffness device parameter λ, will be made by choosing such values that minimize the maximum absolute acceleration. Meantime, it will be checked that the relative structural displacement is be limited.

Two response quantities will be evaluated:

$$\begin{aligned} A &= \frac{\max(a_{a,SAC}(t))}{\max(a_{a,PC}(t))} \\ Y &= \frac{\max(y_{SAC}(t))}{\max(y_{PC}(t))} \end{aligned} \tag{4.3}$$

where A and Y are defined respectively as the maximum values of the absolute acceleration/relative displacement in case of semi active control (SAC) to the maximum corresponding values obtained with classical linear passive control (PC) having assumed a conventional damping ratio of 10%. The response quantities are therefore normalized with respect to the corresponding values assumed in case of conventional linear passive isolation system.

Once having conveniently chosen the algorithm and device parameter ρ and λ respectively, some response functions, all estimated in the stationary response, will be evaluated in order to observe the dynamics of the controlled system and its effectiveness.

The transmissibility factor will be the most important quantity observed. Here, two transmissibility definitions will be utilized, one estimated with the absolute acceleration, TR_a, and the other estimated with the absolute displacement, TR_d, respectively defined as:

$$\begin{aligned} TR_a &= \frac{|a_a(t)|_{max}}{A_g} \\ TR_d &= \frac{|y_a(t)|_{max}}{Y_g} \end{aligned} \tag{4.4}$$

Let's remember that A_g is the acceleration amplitude of the base notion, defined in Eq. 4.1, whereas Y_g represents the displacement amplitude of the base motion.

It is known in literature from problems concerning vibration isolation that transmissibility can be alternatively defined in terms of absolute acceleration or displacement. However, it is important to remark that, only in case of a linear system these two measures are identical. Since the equipment is acceleration sensitive, absolute acceleration transmissibility should be always considered to check the effectiveness of the control strategy. One of the aims of this paper is to investigate what are the main differences between these two representations often interpreted similar in literature. Moreover, the intent is to see if the optimization of the parameters which define the control, leads to the same conclusions in terms of absolute acceleration or displacement transmissibility.

Equipment relative displacement is checked by observing two other quantities. The maximum displacement vibration amplitude, Y_n, that is the ratio of the amplitude of the relative displacement to the static displacement, and the ratio between the maximum relative equipment displacement Y and the maximum amplitude of the input Y_g, named Y/Y_g, defined respectively as:

$$\begin{aligned} Y_n &= |y(t)|_{max} \cdot \omega^2 \\ Y/Y_g &= \frac{|y(t)|_{max}}{Y_g} \end{aligned} \tag{4.5}$$

4.1. Synthesis of the results obtained for the Relative Approach (RA) versus Absolute Approach (AA). In Ref. [9], the authors studied acceleration sensitive equipment isolated with a semi active variable stiffness device with continuous control. Both relative and absolute approaches have been considered. The algorithm and device parameters were conveniently optimized in order to obtain the smallest equipment acceleration in the range of frequencies typical of the isolation.

In has been shown through absolute acceleration transmissibility curves TR_a versus β, that, in the region where typically isolation strategy is considered ($\beta \geq \sqrt{2}$), semi active continuous control gives superior performances in comparison to the case of semi active ON-OFF control and conventional linear passive control (10% of damping ratio). Moreover, comparing relative and absolute approaches, the paper concluded that, absolute approach should be preferred to relative one since the transmissibility seemed to be better controlled, Fig. 4.1. In particular, if the attention was paid in proximity to the resonance condition and in the region around $\beta \geq \sqrt{2}$, the absolute acceleration transmissibility was well limited if compared with relative approach. The two approaches became equal by increasing β. When β=3, reductions of the absolute acceleration transmissibility up to 80% were observed.

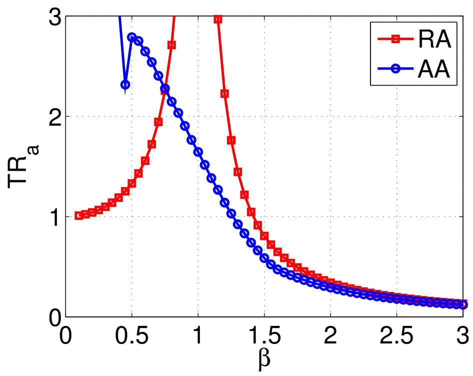

FIG. 4.1. *TR_a varying β for λ=5: continuous SAC, relative approach (ρ=10) and absolute apporach (ρ=0.4).*

5. Results. The aim of this paper is to extend the investigation on the optimal properties of semi active continuous control across the whole frequency spectrum, in order to explore the control system and its effectiveness. The study will focus exclusively on the absolute approach.

The section is organized as follows: first, the optimal parameters ρ and λ at different input frequency ratios β will be selected in order to minimize the absolute acceleration. Then, the absolute acceleration transmissibility curves versus β will be carried out and comparisons among semi active continuous, ON-OFF and passive control will be done. Discussion about utilizing absolute acceleration or displacement transmissibility for acceleration sensitive systems will be debated. Finally, other response functions, defined in Eq. 4.5, to control relative displacement will also be revised. Three regions are meaningful in the frequency spectrum: the region where the input frequency, ω_f, is greater than the system's frequency ω, $\beta > 1$, the region in proximity to resonance condition, β=1, and the region where the input frequency is lower than the system's frequency, $\beta < 1$.

Figure 5.1 depicts the investigation on the properties of the semi active continuous control varying the algorithm and device parameters, ρ and λ, at different values of the frequency ratios β in terms of the dimensionless absolute acceleration and relative displacement response quantities, A and Y respectively. The range

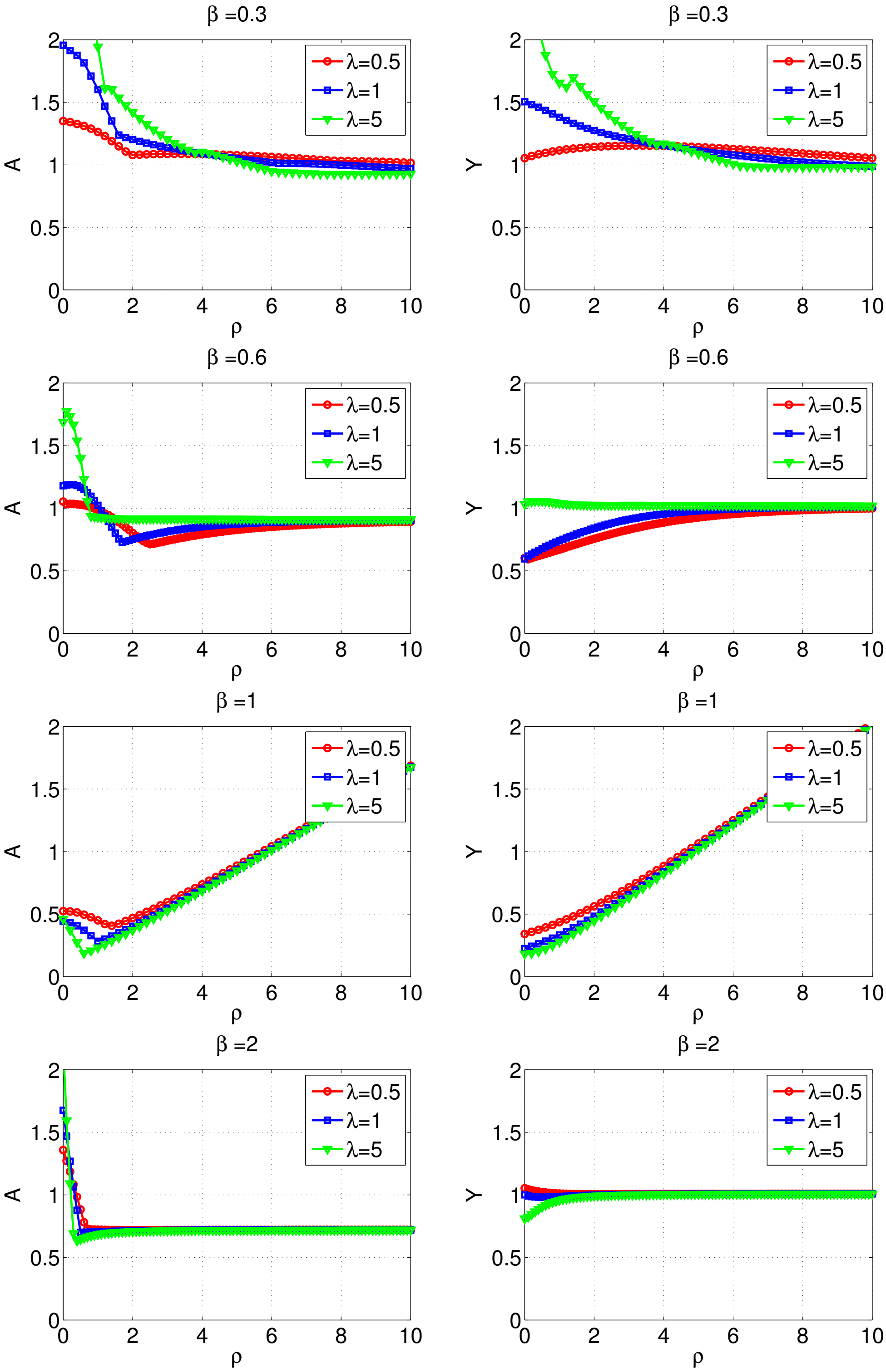

FIG. 5.1. *A and Y varying ρ for different β and λ=0.5, 1, 5.*

of interest of the parameter ρ is among 0 and 10, whereas the λ parameter assumed three values: λ=0.5, 1, 5. Let's observe that, for shortness, λ here means λ_{max}, Eq. 3.5. Optimal ρ and λ are selected as the ones that minimize the absolute acceleration response A.

Since response quantities are dimensionless, when the ordinate values are less than the unity means that response quantities are less respect to conventional linear passive control case. Besides, semi active ON-OFF control corresponds to ρ=0, while semi active continuous control corresponds to $\rho \neq 0$.

Considering the device parameter λ, Fig. 5.1, it can be noticed that, irrespectively of β, the optimal choice is always λ=5. Only in the neighborhood of β about 0.6, the maximum absolute acceleration reduction is obtained for λ=0.5. In fact, for these values of β, the A curves have a well defined minimum at low λ, whereas at λ=5 the curve does not show a minimum and reaches higher values of A.

Considering the algorithm parameter ρ, two different behaviors are observed varying β:

- For $\beta \leq 0.6$, a good choice of ρ can be assuming it at high values, e.g. ρ=10, the effectiveness of semi active continuous control is slightly better than passive control and is always superior to semi active ON-OFF control;
- For $\beta > 0.6$, a good choice of ρ can be assuming it at low values, e.g. ρ=0.7, the effectiveness of semi active continuous control becomes more evident and is always superior to ON-OFF and conventional linear passive control.

By observing the A curves, in proximity to the resonance condition, they have a well defined minimum varying ρ, instead in the other regions the functions decrease until a certain value and then remain constant. Effectively, in [9] an optimal value ρ=0.4 was selected having optimized it only in the region of frequencies typical of the isolation. It can be noticed that the choice ρ=0.7 at $\beta > 0.6$ fits well also in the regions where $\beta \geq \sqrt{2}$, since for such frequencies the acceleration curve is constant increasing ρ (the choice ρ=0.4 or ρ=0.7 is identical in terms of acceleration reduction, see Fig. 5.1 case β=2). However, the frequency ratio zone around β=0.6 where a discontinuity has been observed in the choice of the optimal algorithm and device parameters, deserves a deeper investigation in a future work by the authors in order to explore the peculiar dynamic behavior of the control system.

The relative displacement Y is checked, once selected optimal values of ρ and λ. In general Y is always limited; however, it cannot be identified a systematic trend comparing semi active continuous and ON-OFF control and conventional linear passive control. Relative displacement is not always reduced to the maximum with semi active continuous control.

Figure 5.2 shows absolute acceleration and displacement transmissibility curves, the maximum displacement amplitude Y_n and the displacement ratio Y/Y_g curves versus the frequency ratio β. For comparison purposes, the corresponding curves in the case of semi active ON-OFF control and conventional linear passive control are reported as well. In the case of semi active continuous control the curves are estimated having assumed λ=5, ρ=10 for $\beta \leq 0.6$ and ρ=0.7 for $\beta > 0.6$, whereas, in case of semi active ON-OFF control the curves are estimated for ρ=0 and λ=5.

Since the equipment is acceleration sensitive, absolute acceleration transmissibility TR_a must be primarily observed. Semi active continuous control always leads to better response reduction compared with ON-OFF and passive control. Continuous control can isolate for frequency ratios up to 1.2, but when $\beta \leq 1$ the absolute acceleration transmissibility continues to maintain low values (maximum TR_a is 1.5), which means that the equipment acceleration is sufficiently limited with respect to the base motion. However, it is stressed that acceleration vibration cannot be isolated across the whole frequency spectrum, i.e. absolute acceleration transmissibility cannot be less than the unity across the whole frequency spectrum with any of the assumed three strategies.

Absolute displacement transmissibility curve, TR_d, is different with respect to the absolute acceleration transmissibility curve in the semi active cases; only in case of conventional linear passive control, absolute acceleration and displacement transmissibility are equal.

Therefore, considering absolute acceleration or displacement transmissibility, may lead to different results for evaluating the performance of the control system in the case of semi active control. Displacement vibration can be isolated across the whole frequency spectrum, i.e. absolute displacement transmissibility can be less than the unity across the whole frequency spectrum (semi active ON-OFF control case). By checking the maximum

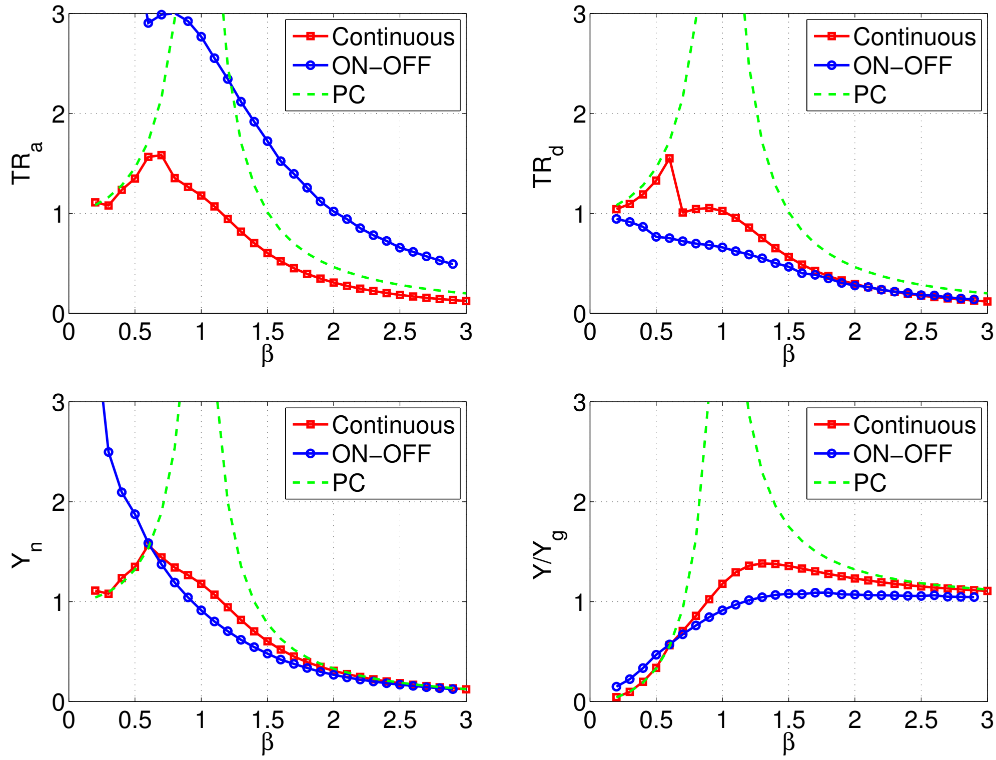

FIG. 5.2. *TR_a, TR_d, Y_n and Y/Y_g varying β for λ=5, semi active continuous (optimized ρ), semi active ON-OFF control (ρ=0), conventional linear passive control, PC, with 10% of damping ratio.*

displacement amplitude Y_n, as $\beta > 1$ it is below the unity only in case of ON-OFF control, when $\beta < 1$ it is greater than one, but still limited only in case of semi active continuous and passive control. By checking the displacement ratio Y/Y_g, when $\beta < 1$, as general trend, it decreases decreasing β and it is almost lower than the unity in the two semi active control strategies, whereas as $\beta > 1$, it tends to the unity as β increases, the convergence is faster in case of semi active ON-OFF control. Curves obtained with semi active continuous and ON-OFF control show light differences, whereas significantly differences are evident with respect to linear passive control, especially in correspondence to the resonance condition.

So far, transmissibility and displacement curves have been obtained having optimized ρ with the frequency ratio, however this action is possible as soon as the input frequency is considered known. If the action and its frequency involved is unknown, such as a natural earthquake, a fixed value for the ρ parameter must be assumed. Figure 5.3 depicts absolute acceleration and displacement transmissibility curves, the maximum displacement amplitude Y_n and the displacement ratio Y/Y_g curves, versus the frequency ratio β for a given value of the ρ parameter. The cases ρ=0.7 (optimal for $\beta > 0.6$) and ρ=10 (optimal for $\beta \leq 0.6$) are reported. For each curve, the results shown in Fig. 5.2, referred to the optimal choice of the algorithm parameter, here can be obtained by crossing the curves at ρ=0.7 and ρ=10. If the expected input action has frequency content mainly in the region over β=0.6 the solution with ρ=0.7 should always be preferred. In fact, acceleration is well limited and displacement is controlled as well.

If the equipment is acceleration sensitive, absolute acceleration transmissibility must be considered and the continuous control law should be preferred to the ON-OFF law or linear passive control. If the equipment is displacement sensitive, absolute displacement transmissibility must be considered and the ON-OFF control law should be preferred to the others.

In order to observe the dynamics of the controlled system with the three strategies, Fig. 5.4 depicts time histories of equipment absolute acceleration for a frequency ratio lower and higher than the resonance frequency

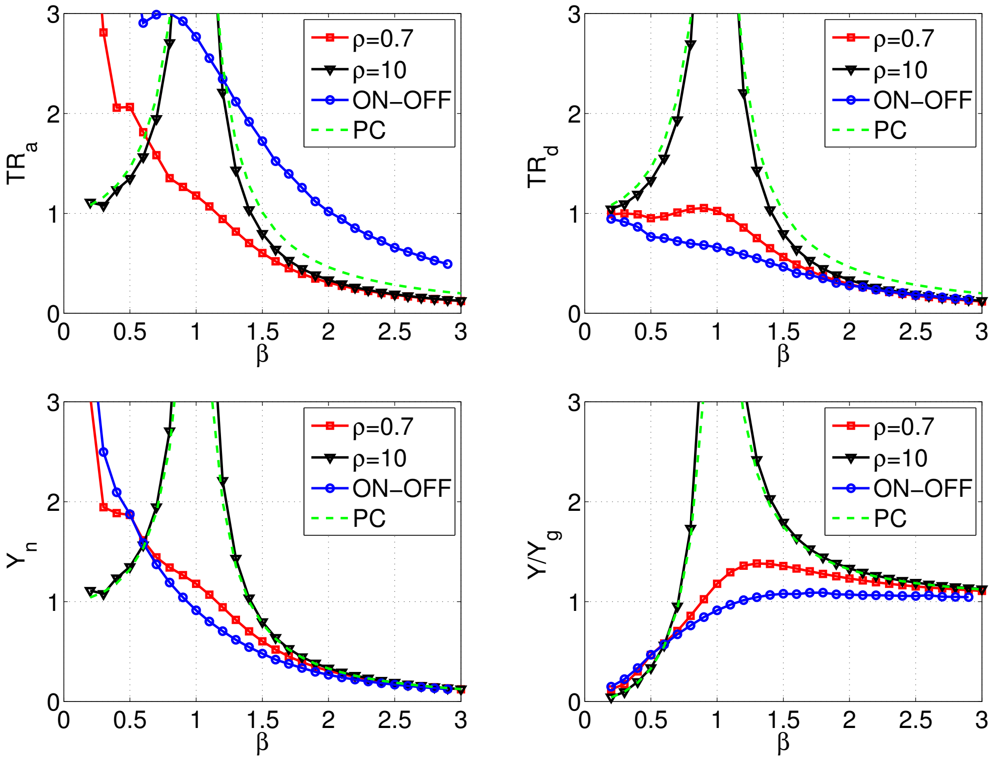

FIG. 5.3. *TR_a, TR_d, Y_n and Y/Y_g varying β for λ=5, semi active continuous (ρ=0.7, ρ=10), semi active ON-OFF control (ρ=0), conventional linear passive control.*

ratio (β=0.8 and β=1.2 respectively), whereas Fig. 5.5 shows, at the same frequency ratios, the variation of the device parameter λ versus time for continuous and ON-OFF control.

The different dynamic behavior in terms of amplitude and shape clearly emerges comparing semi active and passive control; in fact, the acceleration time histories differ from the typical sinusoid of passive control. Semi active continuous control mostly reduces the peaks with respect to the other cases. Semi active ON-OFF case shows the sudden discontinuities in correspondence with the deactivation process, observed in Fig. 5.5 where the time history of the device parameter λ is depicted; instead, continuous case shoots down such peaks in the deactivation process, as a result of the transition in that instants between the maximum and minimum λ being continuous, Fig. 5.5.

By observing the variation of the device parameter λ versus time, Fig. 5.5, in the case of ON-OFF control more than one switch is noticed in one activation and deactivation phase of continuous control for β=0.8, whereas for β=1.2, the frequency of the activation and deactivation phases is almost the same in the two strategies. As known, the transition between the activated (ON) and deactivated (OFF) state is continuous for continuous control and instantaneous for ON-OFF control.

Finally Fig. 5.6 depicts typical absolute acceleration-relative displacement cycles in case of continuous and ON-OFF mode for β=0.8 and β=1.2. A different dynamic behavior in the two cases is observed, and the better effectiveness on the reduction of the absolute acceleration emerges in case of continuous control.

6. Conclusions. This paper treated the topic of base isolation of equipment against vibrations. A SDOF structural model, acceleration sensitive, equipped with a continuously variable elastic device subjected to harmonic input has been discussed, focusing the attention on the absolute approach. The continuous law for the variation of the device parameter has been derived by the Lyapunov method and specialized in order to obtain instantaneous optimality. The aim was to investigate the optimal properties of the semi active continuous control in all the regions of frequency spectrum. Two parameters were optimized in order to minimize the

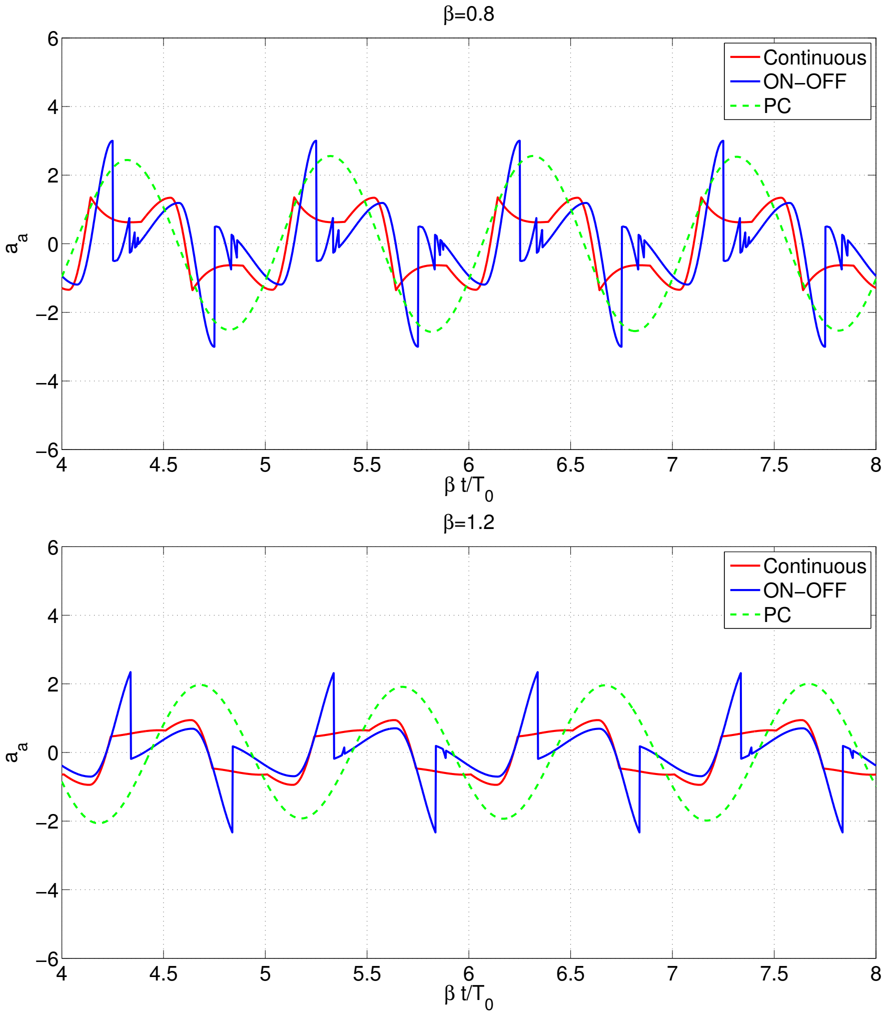

FIG. 5.4. *Equipment absolute acceleration and relative displacement versus time for semi active continuous (ρ=0.7, λ=5), ON-OFF (ρ=0, λ=5), and passive control for β=0.8, 1.2.*

equipment acceleration: ρ related to the algorithm and λ, the stiffness ratio between the device and the equipment, related to the device. It emerged that for the algorithm parameter ρ, two different behaviors are observed varying β: for $\beta \leq 0.6$, a good choice of ρ can be assuming it at high values, whereas for $\beta > 0.6$, a good choice of ρ can be assuming it at low values. For the device parameter λ it was noticed that, irrespectively of β, the optimal choice was almost always to set it at its maximum value in the ON state. This result implies that as the difference among the operational states of the semi active device is greater as the effectiveness of the control system increases. Once optimized the parameters which govern the control system, the absolute acceleration and displacement transmissibility curves, TR_a and TR_d, were evaluated versus β together with the maximum displacement amplitude Y_n and the displacement ratio Y/Y_g. The performance of the continuous semi active control has been evaluated in comparison to semi active ON-OFF and conventional linear passive control.

The performance of the optimized control system was evaluated in terms of absolute acceleration transmissibility curves in order to investigate the behavior across the whole frequency spectrum. It was shown that semi active continuous control always led to better response reduction compared with ON-OFF and linear passive control. It isolated for frequency ratios up to 1.2, still maintaining, for lower frequency ratios, equipment acceleration sufficiently limited with respect to the base motion.

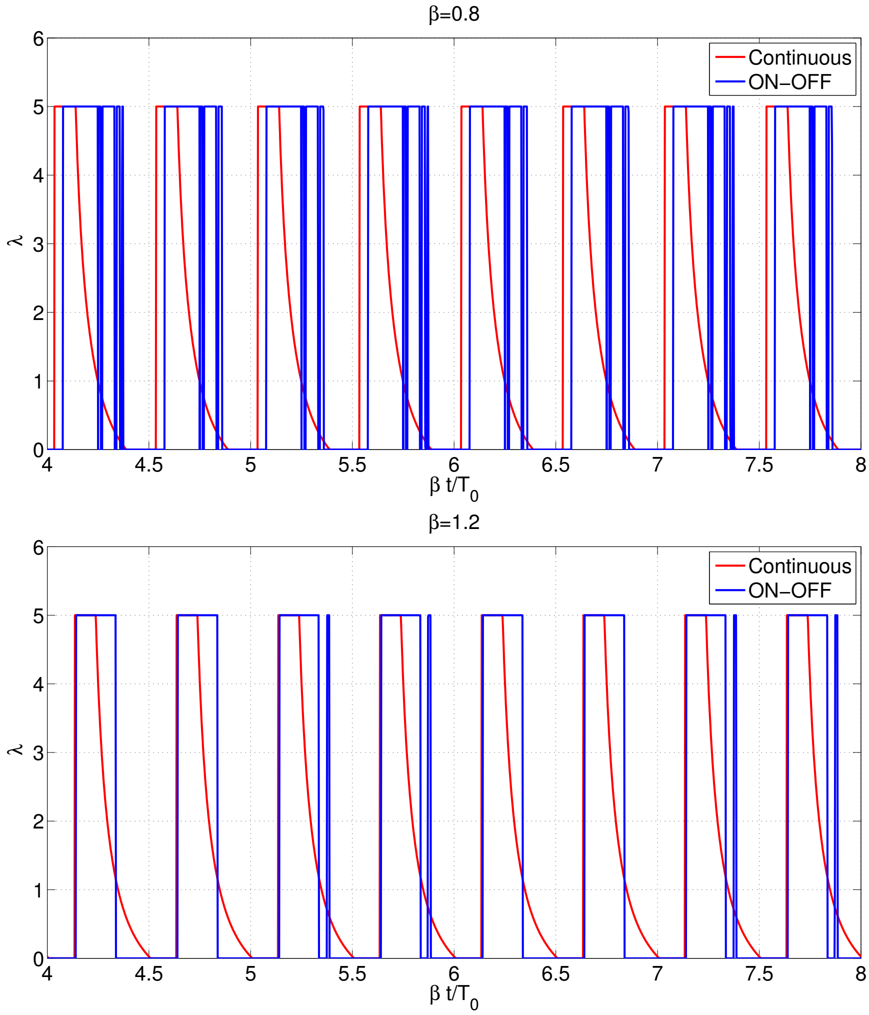

FIG. 5.5. *Variation of the device parameter λ vesrus time for semi active continuous (ρ=0.7) and ON-OFF (ρ=0) control for β=0.8, 1.2.*

In literature from problems concerning vibration isolation, transmissibility can be alternatively defined in terms of absolute displacement. Here, absolute displacement transmissibility curves were estimated as well, in order to investigate if there were differences with the absolute acceleration transmissibility representation. It was observed that absolute displacement transmissibility curves differ with respect to the absolute acceleration transmissibility curves in the semi active cases (continuous and ON-OFF). In fact, only in case of linear passive control, acceleration and displacement transmissibility are equal. Therefore, the performance of a semi active control system may result different if absolute acceleration or displacement transmissibility are alternately considered. Besides, the effectiveness of a control strategy should be always checked with the absolute acceleration transmissibility in case of acceleration sensitive equipment.

Equipment relative displacement was checked by evaluating two other quantities. The maximum displacement amplitude was limited across the whole frequency spectrum only with semi active continuous control. Instead concerning the displacement ratio, only light differences were observed comparing the curves obtained with semi active continuous and ON-OFF control, whereas significantly differences are evident with respect to passive control, especially in correspondence to the resonance condition.

Since the input action and its frequency content is not considered always known, it seems difficult to optimize

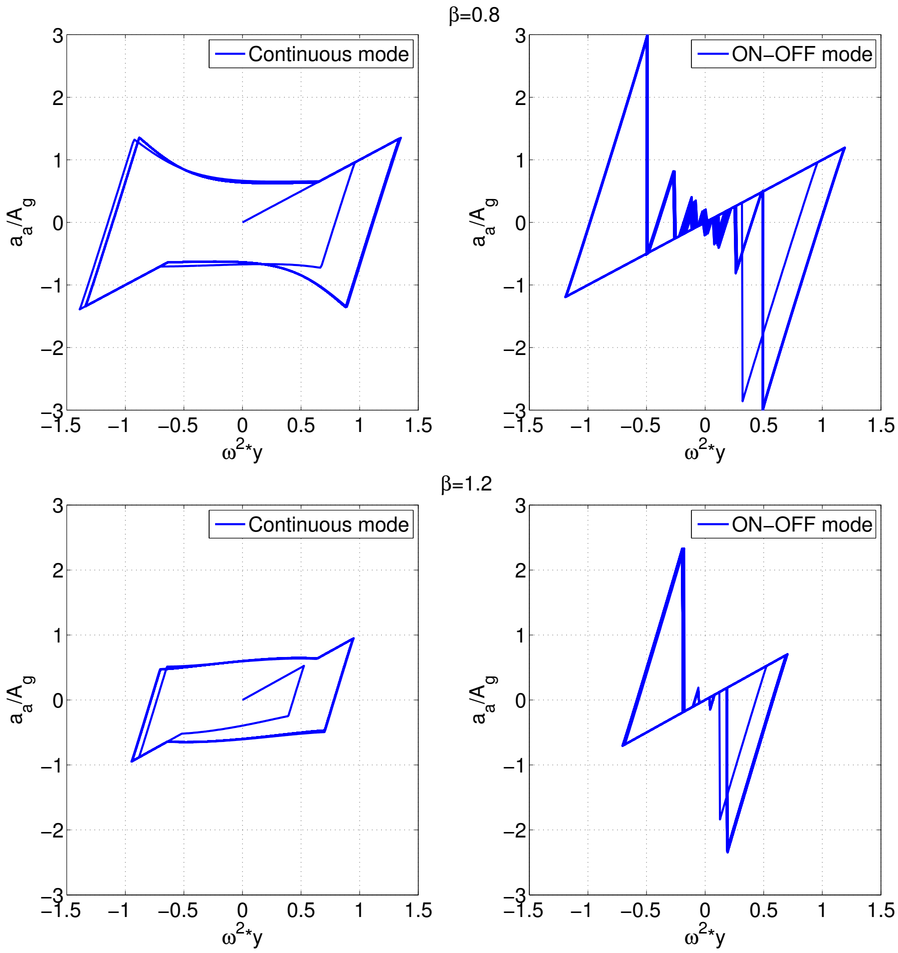

FIG. 5.6. *Absolute acceleration-displacement cycles for semi active continuous (ρ=0.7, λ=5) and ON-OFF (ρ=0, λ=5) control for β=0.8, 1.2.*

the algorithm parameter with the frequency ratio. However, if the expected input action has a frequency content mainly in the region over β=0.6, the solution with ρ=0.7 should be preferred. For equipment acceleration sensitive, the continuous control law gives superior performances with respect to the others: acceleration is well limited and displacement is controlled as well.

In the optimization process, a discontinuity has been observed in the choice of the optimal algorithm and device parameters in the frequency ratio zone around β=0.6. Such region deserves a deeper investigation in a future work by the authors in order to explore the peculiar dynamic behavior of the control system.

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