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APPLICATION OF FINANCIAL MATHEMATICAL MODELS COMBINED WITH ROOT ALGORITHMS IN FINANCE

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Abstract. Investors in the financial markets must deal with various hazards, for which they must create prudent investment portfolios and risk management plans. A multi-objective optimisation approach is proposed using the root algorithm to create a multi-objective root system growth model based on many clusters. An investment risk management optimisation model based on root system growth is built into the study using distributed decision-making. To create a multi-objective root algorithm-based portfolio optimisation model, the Markowitz mean-variance model and a multi-objective root algorithm are employed. According to the findings, the multi-group multi-objective root system method has a real Pareto frontier solution that is more accessible, has a faster convergence rate, and has lower fitness values. The root algorithm's solutions are workable, and the final risk values of 0.0105, 0.0082, and 0.4623 for the 2, 4, and 6 investment objectives are all in the low-risk class range. The optimal set of solutions discovered by the algorithm had better distributivity and convergence. The Hypervolume values for the multi-group multi-objective root algorithm were 5.5298 and 3.9628 for the dual-objective portfolio and the tri-objective portfolio of investment return costs, respectively. The findings of this study can guide the development of portfolio and risk management strategies.

Key words: Financial Mathematics; Portfolio, Risk Management, Root Algorithms, Multi-Objective Portfolios

1. Introduction. A financial portfolio is a collection of various assets combined in a specific proportion with the goal of maximising stability and return on investment (ROI) while minimising risk management (RM). Investors are exposed to risks such as exchange rate risk, market risk, credit risk, and liquidity risk in the financial markets [20]. Investors must create and modify their portfolios differently based on their investment goals, risk tolerance, and other considerations. Financial institutions must employ financial portfolios and RM technologies to manage and mitigate risk, increase operational effectiveness, and boost profitability [13]. This study uses root algorithms (RA) as its foundation. It develops a Multi-Population Multi-Objective Root Algorithm (MPMORA), an Optimization Model for Investment Risk Management Based on Root Growth (OMIRM-RG), and a Portfolio Optimization Model Based on a Multi-objective root algorithm (POM-MORA) to address a portfolio and RM issues in the financial sector. There are four sections to the study. The study of objective optimisation issues in the financial sector and the use of bio-inspired algorithms (BIA) to solve combinatorial optimisation problems are covered in the first section. The second section is based on RA and uses the MPMORA, OMIRM-RG, and POM-MORA constructions. The third section examines the experimental analysis and performance testing of the three models, and the fourth section wraps up the study and identifies its flaws.

2. Related Works. The application of BIA in combinatorial optimisation problems has increased with the growth of bionics and computer science, and some specialists and academics have done studies in this area. To discover and study the occurrence of hazards in the Internet financial market, Qu et al. combined data mining technology and deep learning for processing and analysis. They also presented a radial basis function neural network for ant colony algorithm optimisation. The results of the experiments revealed that this neural network's real error was 0.249, which was different from the goal error of 0.149, demonstrating that the optimisation algorithm may improve calculation results and deliver targeted RM measures [6]. Y Li and colleagues developed a useful design optimisation technique for the symmetry of metamaterial cells. This approach involves the introduction of a cell division mechanism and the development of a new selection mechanism based on this mechanism. The numerical results of the prototype metamaterial cell and the solution of the multi-objective test function showed that the newly proposed method performs better in multi-objective

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optimisation [14]. B R Ke et al. employed particle swarm optimisation techniques, genetic algorithms, and simulated annealing methods to optimise the departure times to lower the building costs and electricity prices of electric buses using a bus system in the Penghu Islands as a study. make sure that each new schedule's price is kept as low as possible without hurting traffic and customer demand. According to research by H, the outcomes demonstrated that departure times could be optimised using the combined strategy, with all costs being lower than the pre-adjustment costs [11]. Wu et al. state that the range of an underwater glider will be directly impacted by how much energy it uses. The parameter values are controlled using a particle swarm multi-objective optimisation technique, which maximises the glider's energy efficiency and location accuracy. Based on a dynamical model of the underwater glider and considering the consequences of unpredictable input errors, a computational model and evaluation procedure for measuring the glider's position accuracy and energy usage are proposed. According to experimental data, the model can boost energy efficiency and improve position accuracy [22].

Since risk is a component of investment activity and no investor or company can be fully risk averse, financial risk management is crucial. Several academics have studied the difficulties with objective optimisation in financial RM. M.A.M. Al Janabi examines the value-at-risk technique's modelling parameters in a market setting and develops a scenario optimisation method for assessing structured portfolios [15]. By assessing available portfolios under operational and financial constraints, experimental results demonstrate that the optimisation algorithm can advance portfolio selection and management in financial markets [2]. A Xie and colleagues take into account a typical risk-averse investor and develop a multi-objective model pricing framework for credit default contracts with periodic payments using a utility non-differential pricing technique. Additionally, buyers of credit default swaps with various trading objectives are investigated to investigate risk aversion's effects on investments. According to the results, credit default swaps might factor in the negative base formation of bonds that followed the financial crisis [23]. The risk preferences of asset liability managers are described by Y Zhang using a hyperbolic absolute risk aversion utility. A generalised drifting Brownian motion with non-Markovian drift and diffusion coefficients defines the cumulative liability process. The best investment plan is constructed by solving a recursively linked stochastic differential equation method using a backward stochastic differential approach. According to experimental data, the model solved the best investment strategy for various parameters [25]. F Chen et al. make the assumption that a financial firm's surplus process is a correlated jump-diffusion process. The insurance company and the financial firm involved in the transaction have a trust issue, and filtering extension techniques are employed to take advantage of the information and alter the insurance business's wealth-creation process. The dynamic mean squared deviation criterion is used to build a bi-objective optimisation model for insurance and investing. The outcomes demonstrated how the model affected the efficient frontier and the equilibrium strategy [3].

In conclusion, BIA is used to solve combinatorial optimisation problems in a variety of domains. However, its use in RM is uncommon. Therefore, this research blends RA with financial mathematical models to address multi-objective optimisation problems in investment RM, which is crucial in investment RM for the financial industry.

3. RA-based Financial Mathematics Model Construction. This section is divided into three parts to construct the model. The first part is based on RA and introduces a multi-objective optimisation strategy to construct MPMORA. The second part is based on RA and uses distributed decision-making to construct OMIRM-RG. The third part is based on a multi-objective RA and constructs POM-MORA based on the Markowitz mean-variance model.

3.1. Construction of MPMORA. . Root Algorithm (RA) is a data structure-based classification algorithm that can divide objects with similar features into the same group. The algorithm's fundamental step is to build a tree structure to represent the objects in the dataset and then iterate through the tree structure to classify the data [11]. The RA algorithm can process ordinary numerical data and handle more complex discrete and structured data. In addition, it is possible to adaptively adjust the tree structure to adapt to different data types and shapes [10]. Since the portfolio problem in financial mathematics is a multi-objective optimisation problem, a multi-objective optimisation strategy is introduced into the underlying RA algorithm to solve it and construct MPMORA. To solve the parameter problem in the model, a fast-dominated sorting method is adopted, and the concept of crowding distance is introduced. Fast undominated sorting stratifies all individuals in the population, assigning an ordinal number to the undominated solution at each level until all individuals are graded [4]. Crowding distance can represent the distance between individuals; the equation is shown in equation (3.1).

$$i p n_j \text{ iff } i_{\text{rank}} < j_{\text{rank}} \text{ or } (j_{\text{rank}} \text{ and } i_{\text{distance}} > j_{\text{distance}} = i_{\text{rank}}) \tag{3.1}$$

In equation 3.1, j_{rank} and i_{rank} denote the non-dominance rank of individuals j and i respectively,j_{distance} and i_{distance} denote the crowding distance of individuals j and i respectively. When selecting particles, preference is given to particles with low non-dominance ranks, and if the non-dominance ranks are equal, the solution with the larger crowding distance is selected. The Pareto optimality criterion (POC) determines the optimal solution [16]. The definition is shown in equation (3.2).

$$\left| P_i(v) = 1 - \frac{S_i(v)+1}{|K'|} \right. \tag{3.2}$$

In equation (3.2) $S_i(v)$ denotes the fast, non-dominated solution ordering method and $|K'|$ denotes the size of the feasible solution. The equation for searching the maximum Pareto frontier solution (PFS) is shown in equation (3.3).

$$\max f(v) = \max\left\{k - \sum_{i=1}^{k} P_i(v)\right\} = \max\left\{\sum_{i=1}^{k}(1-P_i(v))\right\} = \max\left\{\sum_{i=1}^{k}|1-P_i(v)|\right\} \tag{3.3}$$

In equation (3.3), $\sum_{i=1}^{k} P_i(v)$ denotes the sum of all PFS and k denotes the total number of individuals. The individual target values are ranked using Global General (GG), an evaluation model that measures the probability of risk, and all individual target values with variability are first summed, with the equation shown in equation (3.4).

$$GM\left(X_i\right) @ \sum_{X_i \neq X_j} \max\left(\left(\prod_{m=1}^{M} f_m\left(X_i\right) - \prod_{m=1}^{M} f_m\left(X_j\right), 0\right)\right. \tag{3.4}$$

In equation (3.4), X_i and X_j denote two solutions and M denotes the set target number. Thus, having a smaller value of GM dominates the ordering of solutions. The equation for the solution's Global Density (GD) is shown in equation (3.5).

$$GD\left(X_i\right) = \sum_{\substack{j=1 \\ i \neq j}}^{pop} d_{i,j} \tag{3.5}$$

In equation (3.5), $d_{i,j}$ denotes the Euclidean distance between j and i . The solutions are ranked by GM and GD values and the equation is shown in equation (3.6).

$$GG\left(X_i\right) @ \frac{GM\left(X_i\right)}{GD\left(X_i\right)} \tag{3.6}$$

In equation (3.6), $GG(X_i)$ denotes the overall ranking of the solution X_i , indicating that the solution has a better distribution. In multi-population multi-objective RA, all particles are first divided into multiple populations, and individuals in the higher levels are selected using the principle of non-dominated solution ranking. According to the size of the crowding distance, the individuals with large crowding distances are selected to form the set of populations, and the particles are updated several times to obtain the final set of multiple populations. The flowchart of the algorithm for multiple populations with multiple objectives is shown in Fig 3.1.

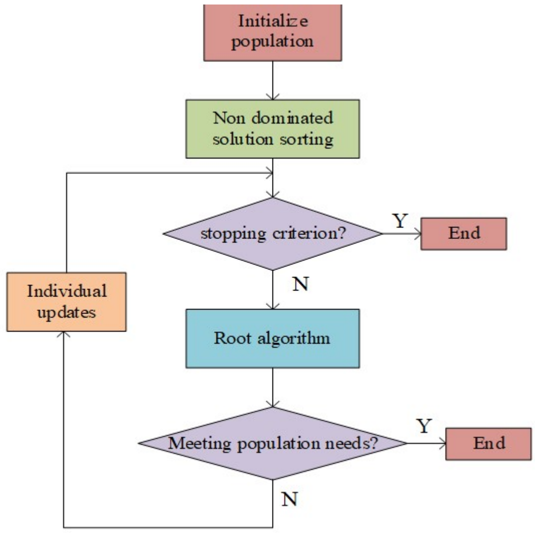

Fig. 3.1: Algorithm flowchart of multiple groups and multi-objective

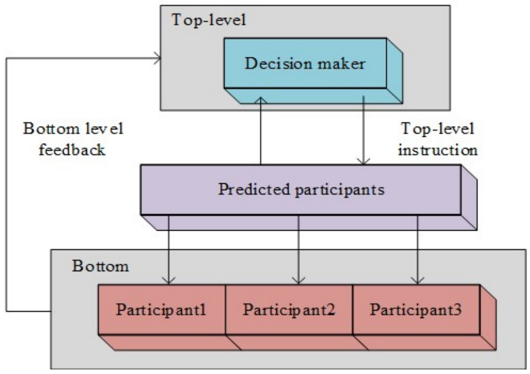

Fig. 3.2: Structure of DDM Decision model

3.2. Construction of OMIRM-RG.. A variety of investment tools and technical tools are frequently required to achieve investment risk management. For example, technical analysis and fundamentals, hedging of portfolios through financial instruments such as financial derivatives, and identification and prediction of investment risks through technical tools such as big data and artificial intelligence [5]. Therefore, distributed decision-making (DDM) is adopted to solve dispersed and complex problems in risk management, and a multi-objective root algorithm model is constructed. The core idea of this algorithm is to transform a multi-objective problem into a root problem, gradually explore the solution space through the root system's growth and branching, and select appropriate branch directions through distributed decision-making. The exchange and collaboration of information between different nodes can help to understand the global problem better and optimize the solutions of each sub-problem through reasonable decision-making. The upper layer model is solved taking into account the characteristics of the lower layer model, and the result of the solution is input to the lower layer and outputted if it meets the requirements, or fed back to the upper layer for re-solving if it

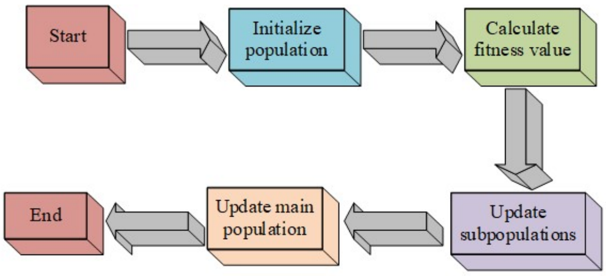

Fig. 3.3: Risk Management Flowchart Based on Root Algorithm

does not. The equation for solving the upper layer model is shown in equation (3.7).

$$\begin{cases} \min \sum_{i=1}^{M} \omega_i R_i\left(B_i\right) \\ \text{ s.t. } \sum_{i=1}^{M} B_i \leq B_{\max} \\ R_i\left(B_i\right) \leq R_{\max}, i=1, \text{ L}, M \\ B_i \in\left[0, K_i\right], i=0,1, \text{ L}, M \end{cases} \tag{3.7}$$

In equation (3.7), B_i denotes the investment funds received by the participant i and B_{max} denotes the maximum investment budget. $R_i\left(B_i\right)$ denotes the level of risk predicted by the decision maker for the participant, R_{max} denotes the maximum level of risk and ω_i denotes the weight. The solution equation for the lower-level model is shown in equation (3.8).

$$\begin{cases} R_i\left(B_i^*\right) \leq \min \sum_{h=1}^{N_i} \sum_{j=1}^{T} u_h f_{h_j}^h\left(x_{h_j}\right) d_j \\ \text{ s.t. } \sum_{h=1}^{N_j} C_{i_h}\left(x_{i_h}\right) \leq B_i^* \\ x_{i_h} \in\left\{0,1, \text{ L}, W_{i_h}\right\}, h=1,2, \text{ L}, N_i \end{cases} \tag{3.8}$$

In equation (3.8), x_{i_h} denotes the optimal strategy chosen by the participant, h denotes the risk factor,N_i denotes the number of risk factors faced by the participant,j denotes the risk level, T denotes the number of risk levels, and $f_{h_j}^h$ denotes the probability of risk occurrence. Since the problem solution of the upper model is a continuous function, while the problem of the lower model is a discrete function, RA is used to construct the distributed decision model [8]. The population is first initialised and the fitness function is used to solve for the fitness value, which is calculated as shown in equation (3.9).

$$\begin{aligned} F_T\left(X_{ik}^T\right) = \sum_{\alpha=0}^{n} w_\alpha R_\alpha\left(X_{i(\alpha+1)k}^T\right) = w_0 R_0\left(X_{ilk}^T\right) + F_B\left(X_{ikk}^L\right) + \phi\left(\sum_{\alpha=1}^{n+1} x_{i\alpha k}^T - \mathrm{I}_{\mathrm{m}\alpha\alpha}\right)^{+} \\ + \eta \sum_{\alpha=1}^{n}\left(\sum_{\beta=1}^{m} \sum_{i=1}^{l} w_\alpha \mu_\beta f_{\beta\lambda}\left(\left|x_{i\alpha\beta kk}^L\right|\right) d_\lambda - R_{\max}\right)^{+} \end{aligned} \tag{3.9}$$

In equation (3.9), $R_0\left(X_{ilk}^T\right)$ denotes the risk level of the decision maker, ϕ and η denote penalty factors, μ_β denotes the weight of the risk factor, d_λ denotes the value corresponding to the risk rating, I denotes the number of risk ratings and $\left(\left|x_{i\alpha\beta kk}^L\right|\right)$ denotes the location index. Based on the solved fitness values, the current optimal particle is determined and the particle population is updated by equation (3.7). Distributed decision-making achieves optimisation of the decision problem through communication and collaboration between the upper and lower models, enabling the whole system to make rapid and efficient decision responses when faced with RM decision problems [12]. Fig 3.3 depicts the RA-based RM flow chart.

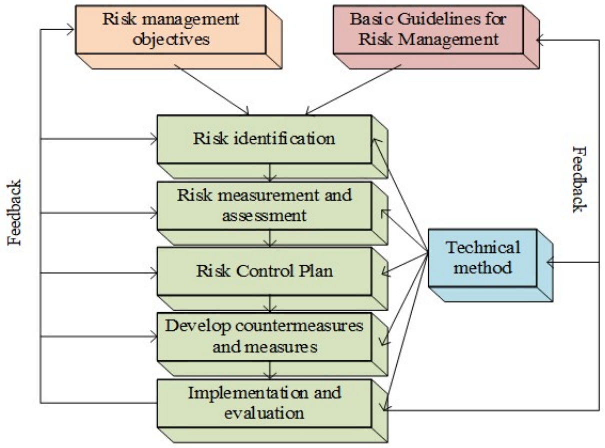

Fig. 3.4: Investment Management Risk Framework for Investors

3.3. Construction of POM-MORA. When constructing a PORTFOLIO, the asset allocation ratio and investment strategy must be determined based on the investor's risk tolerance, risk appetite, investment objectives and other factors [24]. The investor's investment management risk framework is shown in Fig 3.4. Typically, higher-risk investment varieties have relatively higher rates of return but are accompanied by higher risk, while lower-risk investment varieties have lower but more stable returns. The Markowitz mean-variance model was often used to solve the portfolio problem [17]. The Capital Asset Pricing Model (CPAM) is a classic portfolio optimization model mainly used to solve the problem of a single objective: finding a portfolio with minimal risk given the return and covariance matrix of a set of assets. This model is based on two important assumptions: firstly, investors are risk averse and prefer low-risk securities under the same expected returns. Secondly, investors can diversify risk by investing in multiple securities with different risks [19]. The equation for calculating the minimum risk for a given return is shown in equation (3.10).

$$\begin{cases} \min \sigma^2 = x^T W x \\ \max E = u^T x \\ \text{ s.t.e } {}^T x = 1 \quad 0 \leq x \leq 1 \end{cases} \tag{3.10}$$

In equation (3.10), E denotes the sum of expected returns, σ^2 denotes the variance, x denotes a vector of investment weights, W denotes the covariance matrix and e denotes an n-dimensional one-way quantity. The equation for the expected return sum is shown in equation (3.11).

$$E = \sum_{i=1}^{n} u_i x_i \Rightarrow E = u^T x \tag{3.11}$$

In equation (3.11), u denotes the vector of returns and n denotes the type of security. Although the Markowitz mean-variance model laid the foundation for modern portfolio theory, it has shortcomings in practical application [9]. However, when faced with multiple objectives, the Markowitz mean square error model may not be applicable as it cannot directly consider and balance the trade-offs between different objectives [1]. In practical situations, the model results also find it difficult to satisfy a normal distribution, resulting in an inaccurate reflection of actual risks. This model requires investors to provide data on expected asset returns and volatility, but these data have biases that can affect the model's accuracy [18]. Therefore, Markowitz introduced covariance and average return to construct a dual objective investment portfolio. Therefore, Markowitz introduces covariance and mean returns to construct a dual objective portfolio. the equation is shown in equation (3.12).

$$\begin{cases} \min \sigma^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} \\ \max r = \sum_{i=1}^{N} w_i r_i \\ \text{ Subject to } \sum_{i=1}^{N} w_i = 1 \quad 0 \leq w_i \leq 1, i = 1, 2, \text{ L}, N \end{cases} \tag{3.12}$$

In equation (3.12), σ^2 denotes the portfolio, $w_i w_j$ denotes the weights, σ_{ij} denotes the covariance between the two assets, N denotes the number of portfolios and r_i denotes the expected return. In the three-objective portfolio model, in addition to risk and return, there is the objective of minimising expected costs [21]. The semi-covariance is introduced instead of the variance shown in equation (3.13).

$$\begin{aligned} &\left(\frac{1}{T}\right) \cdot \sum_{i=1}^{T} [\text{Min}\,(R_{it} - B, 0) \cdot \text{Min}\,(R_{jt} - B, 0)] \\ &= \sum_{ij\beta} E\,\{\text{Min}\,(R_i - B, 0) \cdot \text{Min}\,(R_j - B, 0)\} \end{aligned} \tag{3.13}$$

In equation (3.13), R_i denotes the return on asset i, B denotes the comparative return, R_{it}denotes the return on asset i at t and R_j denotes the return on asset j . The transaction cost equation is shown in equation (3.14).

$$d\,(W_i, W_j) = \sqrt{\sum_{n=1}^{N} \left(w_n^i - w_n^j\right)^2} \tag{3.14}$$

In equation (3.14), W_i and W_j denote the covariance matrices of asset i and j asset, respectively, W_n^i and W_n^j denote the weights. Thus, the return-risk-cost triple objective portfolio model is represented by equation (3.15).

$$\begin{cases} \text{Max}\, E = \sum_{i=1}^{N} w_i r_i \\ \text{Min}\, R = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sum_{ij} \\ \min C = \sqrt{\sum_{n=1}^{N} \left(w_n^i - w_n^j\right)^2} ji \end{cases} \tag{3.15}$$

In equation (3.15), E denotes the return, R denotes the risk and C denotes the cost. The multi-objective RA is used to solve the multi-objective portfolio problem. The proportional allocation particles for investment management are first initialised to form an initialised population. The fitness value of each particle is calculated and ranked to determine the number of non-dominated layers for each solution [7]. The particles are then selected based on the ranking of the non-dominated solutions and the magnitude of the congestion distance to filter out the unwanted particles and then update the particle population using the initialisation method. Finally, the PFSs are ranked according to the PFS and the solutions with large PFS are output in priority. For investors in the actual use of root growth based investment risk management optimization models, it is necessary to clarify multiple objectives of investors. These goals can include minimizing risk, maximizing returns, controlling transaction costs, etc. Investors should weigh the importance and priority of different goals. In addition to goals, it is also necessary to determine the constraints of investment decisions. These constraints can include funding restrictions, industry restrictions, liquidity requirements, etc. Constraints will help screen feasible investment portfolios. Prepare the data required for investment decisions, which can include historical returns on assets, covariance matrices, asset-related information, etc. Based on the established decision model, run the optimization solution algorithm. Depending on the nature of the algorithm, multiple iterations may be required to gradually approach the optimal solution. Each iteration will generate a new set of investment portfolios until the optimal solution meets the goals and constraints.

4. Model Simulation Experiments and Analysis. This section is divided into three parts to test the model: the first part is MPMORA performance testing and analysis, the second part is OMIRM-RG performance testing and analysis, and the third part is POM-MORA performance testing and analysis.

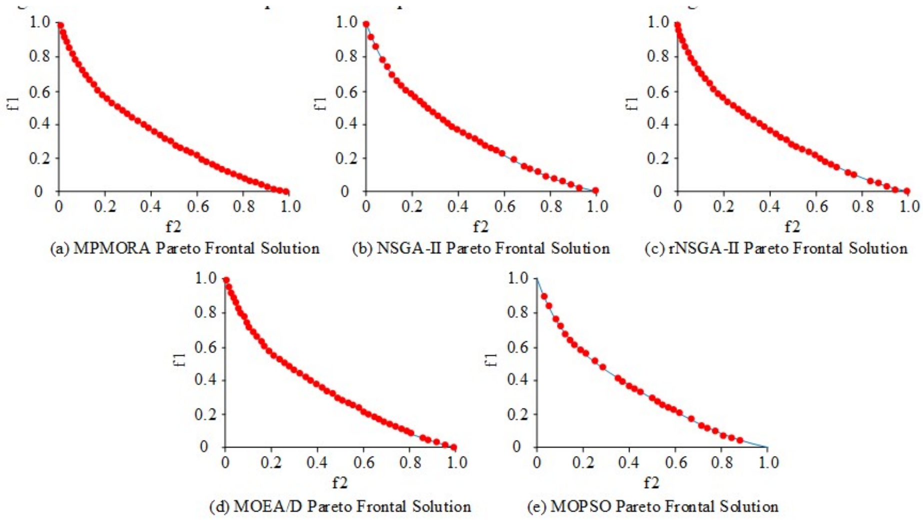

Fig. 4.1: PFS of Five Models on ZDT1

4.1. Performance Testing and Analysis of MPMORA. The experimental test platform was on an Intel core i5 processor and a computer with 8GB of RAM, and the program coding was compiled by the Matlab compiler. The number of populations was set to 200, and the dual-objective test function ZDT1 was used to select the second-generation Non-dominated Sorting Genetic Algorithms- (NSGA-), the second-generation Non-dominated Sorting Genetic Algorithms supporting reinforcement learning (Reinforcement Non- dominated Sorting Genetic Algorithms-II (rNSGA-II), Multi-objective Evolutionary Algorithm Based on Decomposition (MOEA/D), Multi-objective Particle Swarm (MOEA/D), Multiple Objective Particle Swarm Optimization (MOPSO) and MPMORA algorithms were tested for comparison. The experimental results are shown in Fig 4.1.

As can be seen in Fig 4.2, the MPMORA algorithm has a solution closer to the true Pareto front, the rNSGA-II algorithm and the MOEA/D algorithm are the next closest, and the MOPSO algorithm is the worst. The MPMORA algorithm has better performance on the ZDT1 test curve, and the solution results for the two-objective function are closer to the true value. The DTLZ1 three-objective test function and the results are shown in Fig 4.2. The MPMORA algorithm, NSGA-II and rNSGA-II methods, and MOEA/D and MOPSO algorithms are the least consistent with the genuine Pareto solution. The MPMORA algorithm is consistent with the true Pareto solution. Therefore, in the solution solution of the triple objective problem, a fast non-dominated ranking method in RA can be used to present the solution more accurately.

4.2. Performance Testing and Analysis of the OMIRM-RG. To test the performance of RA in investment risk, Genetic Algorithms (GA), Particle Swarm Optimization (PSO), and Artificial Bee Colony (ABC) were chosen as comparisons. The experiments were divided into three different enterprise sizes: decision makers set to one, participants set to two, four and six, and total investment set to 3000. Risk levels were divided into three levels, with a low-risk level for a risk range value of [0.00,0.38], a medium risk level for a risk range of (0.38,0.67 risk, and a risk level of high risk for a risk range of (0.67,1.00]. Fig 4.3 displays the test outcomes. The iteration curves for investors at 2, 4, and 6 are shown in Figures 4.3(a), 4.3(b), and 4.3(c), respectively. The RA method converges with a fitness value of 0.12 in Fig 4.3(a) after 30 rounds. The RA method converges with a fitness value of 0.4 in Fig 4.3(b) after 10 rounds. The RA method converges in Fig 4.3(c) after 10 iterations, with a fitness value of 1.1. Thus, it is clear that RA has a higher testing accuracy and convergence rate. The results of ten tests on the risk values of the four algorithms are displayed in Table 4.1 As can be seen from Table 4.1, the best result for RA at 2 investment objectives is 0.1851, the worst result is 0.2348 and the final value after 10 tests is 0.0105. at 4 investment objectives, the best result is 0.3123, the

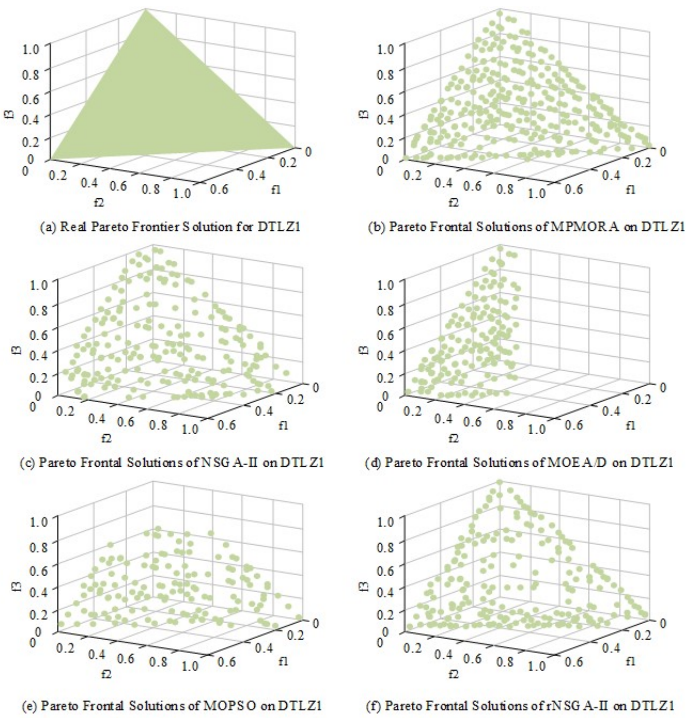

Fig. 4.2: PFS of Five Models on DTLZ1

Table 4.1: Risk Test Results

Number of participants		RA	PSO	ABC	GA
2	Best	0.1851	0.2045	0.2201	0.1825
	Worst	0.2348	0.2895	0.2534	0.2529
	Mean	0.2255	0.2407	0.2456	0.2305
	Standard	0.0105	0.0189	0.0112	0.0589
4	Best	0.3123	0.3167	0.3103	0.2568
	Worst	0.3457	0.4439	0.3772	3.1235
	Mean	0.3186	0.3519	0.3426	0.5029
	Standard	0.0082	0.3019	0.1567	0.5892
6	Best	0.2534	0.3202	0.3719	0.1925
	Worst	1.8623	4.5239	4.0278	2.6438
	Mean	0.5845	0.7598	0.7298	0.8239
	Standard	0.4623	0.8588	0.9236	0.4235

worst result is 0.3457 and the final value after 10 tests is 0.0082. at 6 investment objectives, the best result is 0.2534, the worst result is 1.8623, and the final value after 10 tests is 0.4623. It can be seen that the test values of the RA algorithm end up in the low-risk class range, and the solution provided by the RA algorithm is feasible. The results are plotted in Fig 4.4.

As can be seen from the ANOVA results in 4.4, the RA algorithm and the PSO algorithm have relatively similar variance values for investment targets of 2 and 4, but the RA algorithm shows better robustness for an investment target of 6. Therefore, the RA algorithm performs better and is more robust in dealing with larger-scale problems of risky investment management.

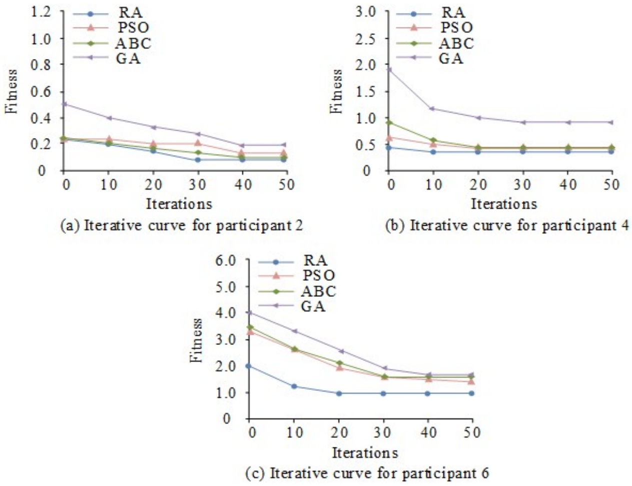

Fig. 4.3: Iteration curves for three different scales

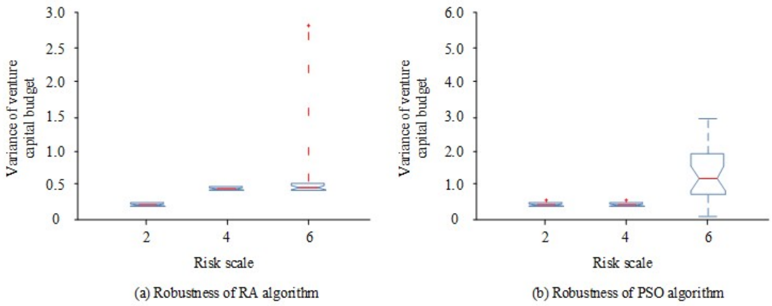

Fig. 4.4: Robustness Analysis Chart

4.3. Performance Testing and Analysis of POM-MORA. The experiment used data from 10 assets of the Beijing Stock Exchange, including monthly rates for each stock from January 2021 to December 2022. The NSGA-II algorithm and the MOEA/D algorithm were selected for testing against the MPMORA algorithm in an investment-return portfolio, the results of which are shown in Fig 4.5. In Fig 4.5, the MPMORA algorithm has better effective curve continuity, the NSGA-II algorithm has the second-best effective curve continuity, and the MOEA/D algorithm has poor effective curve continuity. Therefore, the MPMORA algorithm has a better PFS in the two-objective portfolio, and investors are better able to choose a portfolio approach based on risk appetite. The three algorithms were tested in an investment-return-cost tri-objective portfolio and the results are shown in Fig 4.6. In Fig 4.6, the solutions of the MPMORA algorithm are more homogeneous and diversified, the higher the risk, the smaller the expected return, and the opposite case of cost and risk, cost and expected return. Investors can choose appropriate investment strategies based on their own goals. Through distributed decision-making and information exchange between nodes, investors can better balance the trade-offs between different objectives. For example, while pursuing high returns, attention should also be paid to preventing risks. By exploring a combination of different investment options, investors can find effective solutions to meet multi-objective needs. Table 4.2 displays the test results and compares the Hypervolume values of the three techniques. The MPMORA algorithm's final Hypervolume value is 5.5298 for the two-objective payback combination and 3.9628 for the three-objective payback cost combination, as shown in Table 4.2. Compared

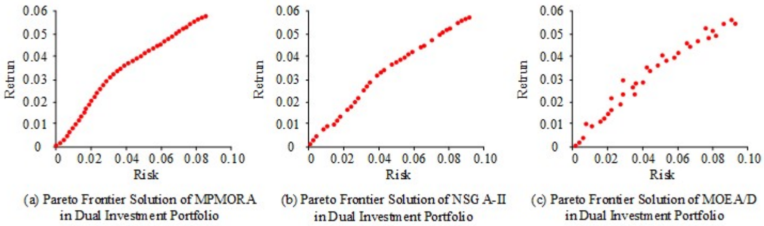

Fig. 4.5: Pareto Curve of Dual Portfolio

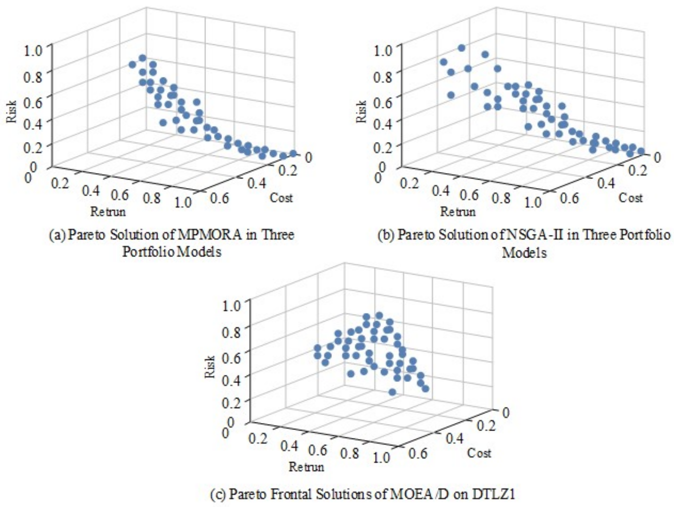

Fig. 4.6: Pareto value of three investment portfolios

to the other two algorithms, the MPMORA algorithm has a smaller Hypervolume value, and therefore, the optimal solution set obtained by the algorithm has better distributivity and convergence.

5. Conclusion. This paper builds MPMORA based on RA, experiments build OMIRM-RG and builds POM-MORA based on the Markowitz mean-variance model to address the multi-objective problem in investment RM. The MPMORA method has a solution that is closer to the real Pareto front, the rNSGA-II algorithm and the MOEA/D algorithm are the next best, and the MOPSO algorithm is the worst, according to the experimental data. The dual objective function solution solutions for the MPMORA method are more accurate and have superior performance on the ZDT1 test curve. After 30 rounds and a 2 investment amount, the RA algorithm converges with a fitness value of 0.12. After 10 iterations and a $4 investment, the RA algorithm converges with a fitness value of 0.4. After 10 iterations and a 6 investment amount, the RA algorithm converges with a fitness value of 1.1. At two investing objectives, RA has a top result of 0.1851 and a worst result of the best outcome for 4 investing objectives is 0.3123, the worst outcome is 0.3457, and the final value is 0.0082. The best outcome for six investment objectives is 0.2534, the worst outcome is 1.8623, and the final value is 0.4623. The RA algorithm's tested values fell within the low risk class range, and the solution it offered was workable. The MPMORA algorithm produced an optimal solution set with improved distribution and convergence, with final Hypervolume values of 5.5298 for the two-objective ROI combination and 3.9628 for the three-objective ROI cost combination. The multi-objective root algorithm based on distributed decision-making can help investors make more informed decisions in multi-objective optimization problems. By decomposing problems, making node decisions, and exchanging information, investors can comprehensively consider different

Table 4.2: Results of Different Portfolio Models

Dual investment target portfolio		MPMORA	NSGA-	MOEA/D	GA
Hypervolume	Max	1.0916	5.8902	3.4527	0.1825
	Min	4.1628	5.192	9.9828	0.2529
	Average	8.3562	5.5362	1.5627	0.2305
	Standard	5.5298	5.9827	7.8392	0.0589
Three investment target combinations		MPMORA	NSGA-	MOEA/D	0.2568
Hypervolume	Max	1.4827	5.4129	6.3862	3.1235
	Min	3.6527	9.8736	7.0382	0.5029
	Average	8.1627	1.0627	2.8617	0.5892
	Standard	3.9628	2.5839	3.9203	0.1925
	Worst	1.8623	4.5239	4.0278	2.6438
	Mean	0.5845	0.7598	0.7298	0.8239
	Standard	0.4623	0.8588	0.9236	0.4235

goals, constraints, and market conditions to optimize their investment portfolios, thereby improving investment effectiveness and risk management capabilities. There are shortcomings in this study. The experimental section selected fewer test parameters and test datasets. In future research, more parameters and experimental data should be selected as the detection objects for testing.

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