



## SIMULATION OF MOTION NONLINEAR ERROR COMPENSATION OF CNC MACHINE TOOLS WITH MULTI-AXIS LINKAGE

XIANYI LI\*

**Abstract.** In order to solve the problem of nonlinear error for a dual rotary table five-axis CNC machine tool due to the linkage of rotary and translational axes, the simulation of motion nonlinear error compensation for a multi-axis linkage CNC machine tool is proposed. The adjacent points in the tool position file are selected as the tool position points for building the model, and then the nonlinear error model resolved by the harmonic function is established according to the error distribution in the classical post-processing. The nonlinear error between the two tool position points is quickly predicted by the analytical expression of this model, and the real-time error compensation of the intermediate interpolation points is realized. Finally, MALTLAB simulation analysis is performed on the tool position file of an impeller part machining to verify the effectiveness of the proposed algorithm. The experimental results show that it can be seen from the distribution curve of the nonlinear error that it is about 10% after compensation as before compensation, thus verifying the effectiveness of the nonlinear error compensation mechanism. The correctness of the nonlinear error analysis and compensation method and the effectiveness of post-processing are verified.

**Key words:** nonlinear error, error compensation, CNC machining, post-processing

**1. Introduction.** Five-axis CNC machine tools are used for machining complex and high-precision surfaces and are widely used in equipment manufacturing, aviation, aerospace and other fields, as the relative motion of the tool and workpiece completes the cutting action. As the machine tool needs to use a large number of discrete linear segments to approximate the contour curve of the workpiece in the process of machining, coupled with the movement of its rotary axis, it will cause inconsistency between linear interpolation and nonlinear motion and generate nonlinear errors, which affects the machining accuracy of CNC machine tools and thus requires effective compensation and control [1]. In NC machining, a series of linear segments are used to approximate the surface of a part linearly. For machining without rotation axis, the tool processes the parts along the piecewise linear trajectory, and the accuracy requirements can be met by setting the tolerance in advance. However, for machining with rotary axes, when the machine tool motion axis to do linear interpolation motion, motion synthesis makes the tool path deviate from the linear segment, forming irregular curves. The error generated at this time is called non-linear error. When processing the curvature of a large change in the free surface, the impact of the error is particularly prominent. Nonlinear errors are difficult to measure and compensate afterwards, but they can be modeled and estimated in advance and pre-compensated to improve the accuracy and quality of the workpiece. The modeling and compensation of nonlinear errors is therefore a key technology in the post-processing of CNC machining [2].

Five-axis machining greatly expands the machining capability and range of machine tools and significantly reduces repositioning errors. Five-axis machine tools play an irreplaceable advantage in the field of impeller blade, complex surface, mold development, etc. Because of the flexible tool attitude control, domestic and foreign research and machine tool manufacturers have conducted many aspects of five-axis machine tool research, the kinematic analysis of the machine tool orthogonal structure has been basically perfect, but in the five-axis machine tool non-linear error research, there are many aspects need to be improved [3]. The rotary axis of 5-axis CNC machine tools can change the direction of the tool axis according to the machining requirements, which can improve the machining efficiency of the machine and reduce the number of clamping of the parts. However, the structure of five-axis machine tools is complex, so there are many technical difficulties in the actual machining. Based on the nonlinear errors generated by the five-axis linkage and the structural characteristics

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\* School of Mechatronics and Energy Engineering, NingboTech University, Ningbo, Zhejiang, 315100, China (Corresponding author, [XianyiLi6@163.com](mailto:XianyiLi6@163.com))

of the dual rotary table five-axis machine, the main research of this study is to establish the compensation method and kinematic model of this type of machine [4].

In the multi-axis CNC machining of complex surfaces, due to the influence of machine tool rotary axis motion, the tool trajectory of adjacent tool points is not an ideal interpolated straight segment, but a spatial curve connecting the straight segment, and the difference between them is called nonlinear error. The existence of nonlinear errors can seriously affect the machining accuracy of the surface, so scholars at home and abroad have conducted many studies on the analysis and compensation of nonlinear errors.

**2. Literature review.** Nonlinear error is a common problem in the five-axis linkage CNC machining process. The reason is that the five-axis linkage CNC system is not continuous trajectory control, but discrete point control. Based on the processing accuracy requirements, CAM software generated tool position file scatters the free curve into tiny linear segments, but due to the five-axis CNC machine tool rotation axis (such as table rotation or tool swing) to join, the trajectory between adjacent tool position points in the actual machining process is a spatial curve instead of a straight line, thus causing a nonlinear error. Therefore, the analysis and compensation of nonlinear error is needed for multi-axis CNC machining [5]. In-depth analysis of the distribution characteristics of this error, two nonlinear error compensation methods based on the workpiece coordinate system and the machine coordinate system are proposed, namely the front nonlinear error compensation method and the back nonlinear error compensation method. These two error nonlinear error compensation methods are to compensate the error in advance and improve the machining quality of the parts. The front nonlinear error compensation method first obtains the posterior processing points of the machine tool coordinate system according to the known tool position points in the workpiece coordinate system, while sampling uniformly in both coordinate systems [6]. Then the post-processing of the sampled points in the workpiece coordinate system and the sampled points in the machine coordinate system are used to construct a compensation vector, which is pre-processed and calculated to obtain the error of the uniformly sampled points, and finally the pre-nonlinear error compensation method is constructed from the sampled points and the tool position points to predict the ideal tool curve trajectory. The method interpolates the curve tool trajectory in the work coordinate system, and then obtains the interpolation points in the machine coordinate system by the classical post-processing, so as to reduce the nonlinear error [7]. The posterior nonlinear error compensation method first selects two points in the tool position file as the tool position points for establishing the compensation method, and then establishes the posterior nonlinear error compensation method according to the maximum error distribution in the middle of the two tool position points in the classical post-processing, and finally the analytical function equation of this compensation method is used to quickly predict the nonlinear error between two adjacent points in the tool position file, so as to realize the real-time error compensation of the intermediate interpolation point.

Five-axis linkage machine structure is complex, with more types of machine tools. It is basically in the three-axis machine tool based on the addition of two rotary axes. The machine is equipped with different control systems. The tool position files generated in the post-processing system comes with CAE or CAD software may not be directly used in CNC machining. It needs to use the post-processing system to convert the tool track file into a specific machine can identify the CNC machining code. Therefore, post-processing has an important role in multi-axis machining [8].

At present, foreign high-grade CNC systems provide a rotary tool center point programming function, which aims to reduce nonlinear errors and improve machining accuracy. However, the core algorithms of these functions are not disclosed. For the characteristics of dual rotary table five-axis CNC machine tools, this study establishes the analytical expression of the nonlinear error model of this type of machine tools, and further proposes a nonlinear error compensation mechanism with RTCP compensation effect. Finally, the effectiveness of this compensation mechanism is proved by MAT-LAB simulation analysis with the impeller as the simulation model.

### 3. Methods.

#### 3.1. Dual rotary table 5-axis machine post processing and non-linear errors.

**3.1.1. Dual rotary table five-axis CNC machine tool machining process.** The general five-axis machining process involves CAM software generating a tool position file in the workpiece coordinate system, post-processing it to obtain the corresponding numerical control code (NC code), and then loading the machine

from the NC code for machining. The points in the Workpiece Coordinate System (WCS) are then converted to Machine Coordinate System (MCS) points after compensating and post-processing.

**3.1.2. Dual rotary table five-axis CNC machine tool reverse motion model.** The reverse motion model of five-axis CNC machine tool with double turntable is established, namely, the coordinate values in the workpiece coordinate system are known, and the translation quantity of the three translation axes and the momentum of the two rotation axes of the machine tool are solved [9].

$$\begin{aligned}
 c\theta_A &= \arccos\theta\left(\frac{u_z}{L}\right); \\
 \theta_C &= \arctan\left(\frac{u_x}{u_y}\right); \\
 S_x &= \cos\theta_C \cdot (P_x - m_x) - \sin\theta_C \cdot (P_y - m_y) + m_x; \\
 S_y &= \sin\theta_C \cdot \cos\theta_A \cdot (P_x - m_x) + \cos\theta_A \cdot \\
 &\quad \cos\theta_C \cdot (P_y - m_y) - \sin\theta_A \cdot (P_z - m_z) + m_y; \\
 S_z &= \sin\theta_A \cdot \cos\theta_C \cdot (P_x - m_x) + \sin\theta_A \cdot \cos\theta_C \cdot \\
 &\quad (P_y - m_y) + \cos\theta_A \cdot (P_z - m_z) + m_z;
 \end{aligned} \tag{3.1}$$

where  $\theta_A$  denotes the angle of rotation of the axis A;  $\theta_C$  denotes the angle of rotation of the axis C;  $P_x, P_y, P_z$  denotes the translation distances of the axis X, the axis Y and Z.

**3.2. Nonlinear error modeling and compensation.** In this study, the harmonic function is selected as the basis of the polynomial according to the distribution characteristics of the nonlinear error. In order to simplify the function model, the first-order polynomial is selected for modeling, while the mathematical analytic parameters of this model are determined according to the nonlinear error data obtained from simulation experiments, so as to establish the mathematical model of this error.

**3.2.1. Non-linear error compensation algorithm.** The purpose of error compensation is to find the interpolation point corresponding to the point with the maximum permissible value of error on the nonlinear error model, and then further obtain other locations where the data points need to be reinserted for error compensation. The expression of the nonlinear error model is taken as the compensation function, and the compensation point of the error can be easily found [10].

Considering that the maximum value of error calculated according to the model can truly reflect the accuracy of the workpiece during actual machining, while the maximum allowable value of error characterizes a maximum limit of the accuracy of the machined part, if it exceeds this range, the workpiece may not meet the design requirements and cannot be used, so the basic rules of the nonlinear error compensation algorithm are given in this study.

The magnitude of the maximum value of the error calculated from the model is compared with the maximum allowable value of the error. If the maximum value of the model is larger than the maximum allowable value of the error, the nonlinear error compensation is performed, otherwise, no compensation is performed [11].

**3.2.2. Nonlinear error modeling process.** The above analysis establishes the harmonic function model and nonlinear error compensation mechanism to predict and compensate the nonlinear error at the interpolation point, and the specific process is shown in Figure 3.1, which is divided into three modules, namely, the post-processing module, the nonlinear error modeling and compensation module, and the nonlinear error calculation and comparison module.

The dashed box in the figure is the general post-processing module, which firstly interpolates the tool position points linearly, and then substitutes the interpolated points into the inverse kinematic model of the dual rotary table five-axis CNC machine tool proposed in this study to obtain the classical post-processing points, that is, the coordinate points and tool axis vectors in the workpiece coordinate system are converted into the coordinate points in the machine coordinate system and the angular displacement of the two rotary axes; the solid box is the modeling and compensation module of nonlinear error. In this study, a real-time prediction nonlinear error model is established and compensated; the dotted box is the nonlinear error calculation

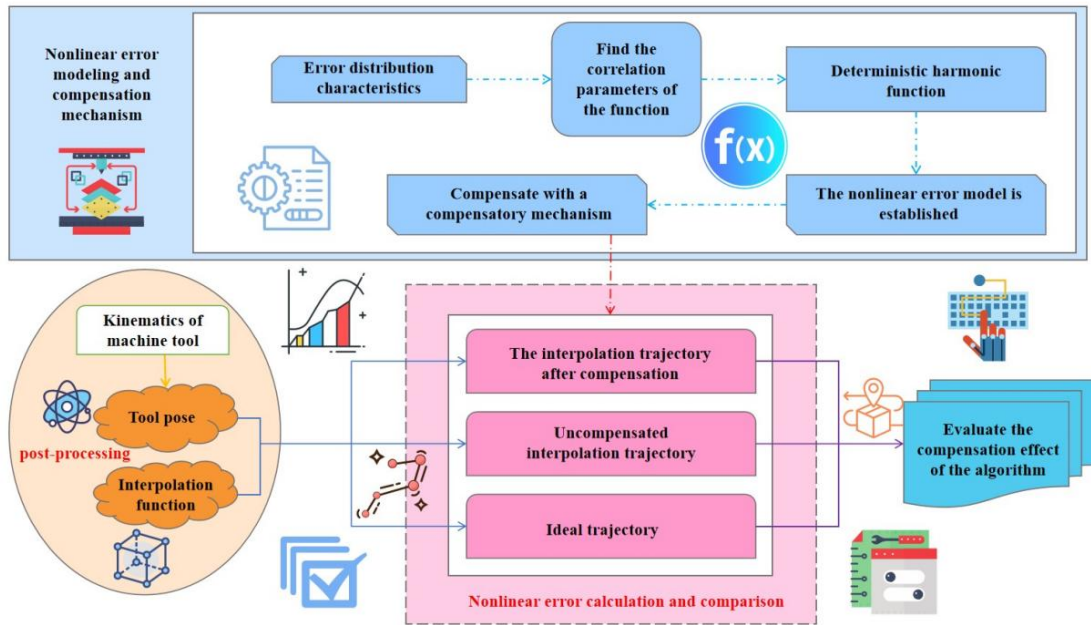


Fig. 3.1: Error modeling and compensation flow

Table 3.1: Interpolation point nonlinear error  $\varepsilon$  mm

Number	1	2	3	4	5	6	7	8
$\varepsilon$	0	0.1699	0.3235	0.4608	0.5819	0.6867	0.7752	0.8473
Number	9	10	11	12	13	14	15	16
$\varepsilon$	0.9032	0.9428	0.9660	0.97290.9635	0.9378	0.8957	0.8373	
Number	17	18	19	20	21	22	23	
$\varepsilon$	0.7626	0.6716	0.5643	0.4407	0.3009	0.1447	0	

and comparison module, which compares the nonlinear errors of different trajectories, mainly comparing the interpolated trajectory and numerical values, the interpolated trajectory part is to compare the compensated interpolated trajectory, the uncompensated interpolated trajectory and the ideal trajectory in the machine tool coordinate system, and the numerical part is to compare the compensated trajectory and the uncompensated trajectory on the two-dimensional graph. The numerical part is to compare the compensated trajectory and the uncompensated trajectory on a two-dimensional plot [12].

**3.2.3. Substitution and post-processing of tool position points.** The parameters of the tool point (all in mm) are as follows.  $P_0 = (-191.4037, 11.3508, 7.6182)$ ,  $U_0 = (0.00639, -0.036377, 0.99927)$ ,  $P_1 = (-191.4057, 13.5159, 7.9093)$ ,  $U_1 = (0.006393, -0.068922, 0.99960)$  where P is the tool position in the workpiece coordinate system and U is the tool axis vector in the workpiece coordinate system. Linear interpolation is used to substitute these interpolation points into equation 3.1 to find out the corresponding post-processing points. In this study, the ideal trajectory is approximated as a straight line, and the nonlinear error is the deviation of the actual tool position trajectory to the straight line. The calculated nonlinear errors are shown in Table 3.1, which gives 23 interpolation points and their corresponding nonlinear error values. Figure 3.2 shows the distribution of the nonlinear error values of the interpolation points, where the horizontal coordinate is the interpolation point in the middle of  $P_0$  to  $P_1$  and the vertical coordinate d is the value of nonlinear error.

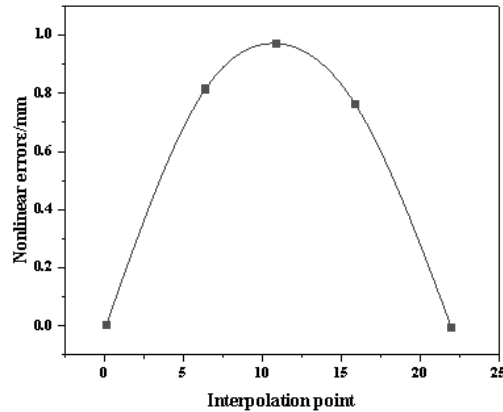


Fig. 3.2: Non-linear error distribution

**3.2.4. Harmonic function modeling.** According to the characteristics of the nonlinear error distribution, the approximation is based on the harmonic function, and the specific expression is

$$P(x) = \sum_1^m A_j \cdot \sin(kx) + B_j \cdot \cos(kx), m = 1, 2, 3, \dots, n_0 \tag{3.2}$$

Where  $A_j, B_j$  is the amplitude of the harmonic function and  $m$  is the order of the polynomial. In this study, the first-order polynomial is used to approximate the nonlinear error for simplicity of calculation. The error at the first and last two points is zero, and the initial value is  $b_j = 0, m = 1, x = \pi$ . Then equation 3.3 can be reduced to

$$\begin{aligned} A_j &= P'_{mid} - \frac{(P'_1 + P'_0)}{2}; \\ k &= \frac{x_i - x_0}{x_1 - x_0}; \\ P(x) &= A_j \cdot \sin(k \cdot \pi). \end{aligned} \tag{3.3}$$

Where  $A_j$  is the amplitude of the nonlinear error function, and the amplitude of the harmonic function is selected as the error at the midpoint, because the maximum value of the nonlinear error is distributed in the middle of the knife position point, so  $A_j$  is the ideal linear midpoint  $(P'_0 + P'_1)/2$  to the midpoint of the post-processed is the vector of This is the relative position of the ULR point, and the nonlinear error at the ULR point can be derived from the corresponding position [13].

**3.2.5. Nonlinear error harmonic function compensation mechanism.** In this study, data matrices are constructed in MATLAB, and these matrices are used to store the three-dimensional data of the post-processing points after the nonlinear error compensation. The compensation function  $\varepsilon$  is  $P(x)$ , and the harmonic function error model can be obtained from equation 3.3. The distribution of nonlinear errors is predicted according to the nonlinear error model for nonlinear error compensation, and the analytical formula is

$$\begin{aligned} \varepsilon &= P(x) \\ P_\lambda'' &= P'_\lambda - \varepsilon_0 \end{aligned} \tag{3.4}$$

The matrix is derived by equation 3.3  $P_x^t$  of the error compensation value, and substitute into equation 3.4 to get the post-processing point after compensation. The residual nonlinear error between the uncompensated

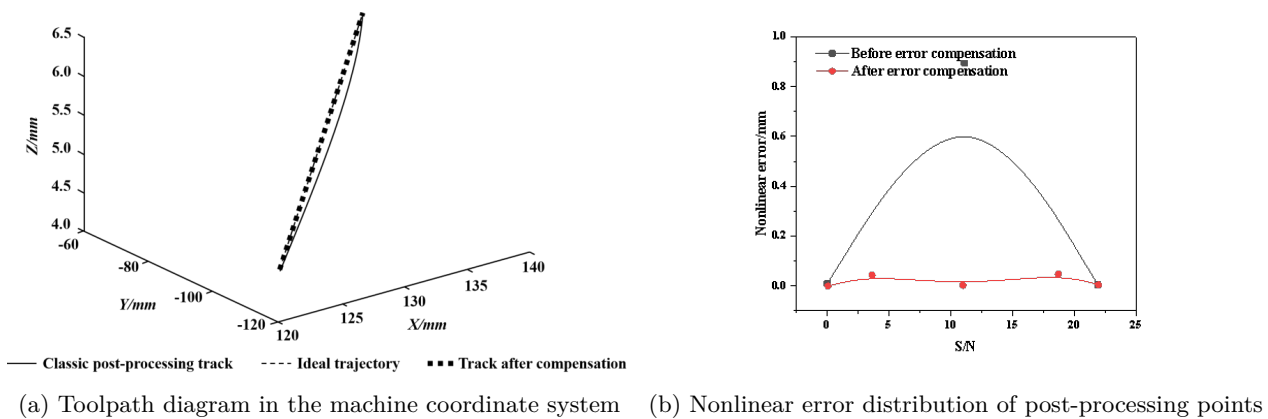


Fig. 3.3: Distribution of tool trajectory and non-linear error at the verification point

and this compensation are compared, as shown in Figure 3.3a, the forked dotted line is the uncompensated tool trajectory, the dotted line is the compensated tool trajectory, and the solid line is the ideal trajectory; the dashed line in Figure 3.3b is the uncompensated tool trajectory error, and the star dotted line is the compensated tool trajectory error [14].

**3.3. Data simulation and experimental validation.** A typical 5-axis machined part, an impeller, was selected for verification. The impeller's blades are free-form, so multi-axis machining is required in The multi-axis machining mode generates the machining tool position file of the workpiece. In this study, a section of the main blade machining tool position file of the impeller is selected for verification [15].

The machining mode is used to export the data of the entire machining tool position file  $P(x, y, z)$  is the spatial coordinate of the tool point,  $U(i, j, k)$  is the spatial coordinate of the tool axis vector. A section of tool position file with 20 tool points is selected for verification, as shown in Table 3.2.

MATLAB is used to calculate all intermediate interpolation points, and the post-processing points are obtained by post-processing. The nonlinear error of each post-processing point due to rotary axis linkage is calculated by the harmonic function model of nonlinear error, and then the nonlinear error compensation mechanism proposed in this study is used to compensate the error of the post-processing points [16].

**4. Results and Discussion.** The tool position points in Table 3.2 are linearly interpolated and post-processed (Eq. 3.1) to obtain the coordinate information of the five axes of the machine and the position of the interpolated trajectory in the machine coordinate system. Figure 4a shows the comparison of tool trajectory, where the dashed line is the curve without compensation and directly post-processed, and the solid line is the trajectory after compensating the error with the nonlinear error compensation function proposed in this study, and the dotted line is the ideal trajectory. Figure 4.1b shows a partial enlargement of Figure 4.1a, which shows that the solid line is closer to the dotted line than the dashed line, i.e., the curve using the proposed nonlinear error compensation method is closer to the ideal trajectory than the uncompensated curve [17].

The distribution curve for the nonlinear error shows that it is about 10% after compensation as before compensation, thus verifying the effectiveness of the nonlinear error compensation mechanism.

In order to address the nonlinear errors generated by the linkage of rotary and translational axes in a dual rotary 5-axis CNC machine tool, this study proposes an analytical model to predict and compensate for the nonlinear errors in real time. The first-order harmonic function is selected for fitting, and the analytical expression of the distribution of the harmonic function error model is established with the first and last two points and the middle point with the characteristic that the maximum value of this error is distributed at the middle point [18].

The simulation results show that the nonlinear error vector compensation according to the harmonic func-

Table 3.2: Points in 20 tool position files

Number	Tool position coordinates			Tool axis vector		
	P(x,y,z)			U(x,y,z)		
1	51.4984	4.8320	-37.8134	0.8259	-0.0720	0.5592
2	50.6826	4.2881	-37.3166	0.8315	-0.0748	0.5505
3	49.8963	3.7208	-36.7989	0.8370	-0.0776	0.5418
4	49.1479	3.1214	-36.2629	0.8423	-0.0804	0.5330
5	48.4270	2.5096	-35.7040	0.8476	-0.0832	0.5241
6	47.7376	1.9901	-35.1263	0.8527	-0.0860	0.5152
7	47.0816	1.2247	-34.5395	0.8578	-0.0889	0.5062
8	46.4617	0.5413	-33.9464	0.8628	-0.0918	0.4972
9	45.8699	-0.1495	-33.3338	0.8676	-0.0947	0.4882
10	45.3113	-0.8505	-32.7022	0.8724	-0.0976	0.4790
11	44.7915	-1.5774	-32.0679	0.8770	-0.1005	0.4699
12	44.3058	-2.3069	-31.4100	0.8815	-0.1035	0.4606
13	43.8511	-3.0438	-30.7382	0.8860	-0.1064	0.4514
14	43.4218	-3.8037	-30.0768	0.8903	-0.1094	0.4420
15	43.0247	-4.5475	-29.3775	0.8945	-0.1124	0.4327
16	42.6480	-5.3006	-28.6769	0.8986	-0.1154	0.4232
17	42.2853	-6.0689	-27.9865	0.9027	-0.1184	0.4138
18	41.9514	-6.8326	-27.2762	0.9066	-0.1214	0.4043
19	41.6252	-7.6173	-26.5869	0.9103	-0.1244	0.3947
20	41.3061	-8.4195	-25.9157	0.9140	-0.1274	0.3851

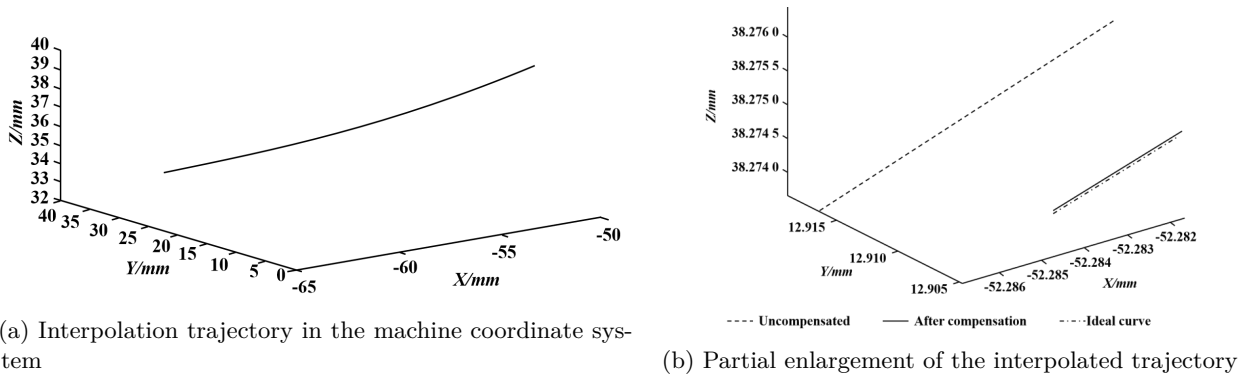


Fig. 4.1: Toolpath comparison chart

tion model can effectively improve the error magnitude, and the peak of the nonlinear error compensation using the nonlinear error harmonic function compensation mechanism does not exceed 0.5, and the peak of the uncompensated trajectory error exceeds 4.5, and the error is effectively improved [19,20].

**5. Conclusion.** In this study, a simulation for compensation of nonlinear error of multi-axis linkage of CNC machine tool motion is proposed. The nonlinear error model established by using harmonic function investigated in this study in the machine tool coordinate system needs to calculate the coefficients of the harmonic function. For the nonlinear error generated by the linkage of rotary and translational axes in a dual rotary five-axis CNC machine tool, an analytical model for real-time prediction and compensation of nonlinear error is proposed. The first-order harmonic function is selected for fitting, and the analytical expression of the distribution of the harmonic function error model is established with the first and last two points and the

middle point with the characteristic that the maximum value of the error is distributed at the middle point. The nonlinear error vector compensation based on the harmonic function model can effectively improve the error size.

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