



## A SIMD ENVIRONMENT FOR GENETIC ALGORITHMS WITH INTERCONNECTED SUBPOPULATIONS

DEVARAYA PRABHU\* AND BILL P. BUCKLES† AND FREDERICK E. PETRY†

**Abstract.** The algorithmic form of GAs conforms well to SIMD computing environments with relatively minor adjustments to the operators. In this paper we consider in detail a GA implementation on a MasPar machine. The question of the degree to which control parameters affecting intercommunication impact performance is addressed using ANOVA methods. The purpose is to supplant anecdotal experience with statistical evidence. A set of control parameters—topology, migration operator, migration radius, and migration probability—were chosen together with four representative levels of each. Metrics for three response variables—efficiency, diversity, and schema propagation—were developed that allowed insight into the behavior under the various parametric conditions. These were incorporated into three  $4 \times 4 \times 4$  randomized factorial experiment designs. Among other things, it was determined that the interconnection topology is not in itself a significant factor but the extent of connectivity and frequency of communication are. An important outcome of this study is that, while the individual factors are significant, the factors do not interact in unexpected ways.

**Key words.** Genetic algorithms, massively parallel computation, communicating subpopulations, migration, Royal Road functions, experiment design.

**1. Introduction.** Because of the complexity of large optimization problems, the value of parallel realizations of optimization algorithms has become quite evident [10, 11]. In particular, in this paper we present a study of an approach to parallel genetic algorithms.

Amenability to parallelization has always been viewed as a major strength of Genetic Algorithms (GAs). GAs operate on populations of individuals. When the population is distributed into (perhaps overlapping) subpopulations, the introduction of new operators is possible as well as altering the semantics of existing ones. We consider a GA to be parallel if its operators reflect the new interactions that are enabled by the presence of subpopulations.

Furthermore, we are interested in the case for which there are thousands, not dozens, of subpopulations. When there are a massive number of subpopulations, there are larger number of local behaviors competing for the opportunity to expand and their expansion is more acutely observable. The operators that affect subpopulation interactions are defined in part by parameters. In some cases, the number of alternative values for these parameters increase as the number of subpopulations increase. Thus choices made by algorithm designers must be more informed. These two factors, measurability of effects and the need for insights, make massively parallel GAs a useful domain of investigation.

One aspect of this paper is the methodology that is applied, namely, experimental design methods [21, 29]. It has the purpose of establishing the quantitative significance of various factors, singly and in combination, that affect performance. This is in contrast to the “typical” GA experiment that quantifies performance prediction but only qualifies the factors affecting the prediction. The method used here, we believe, should be used more often because, beyond quantification, it is able to elicit evidence of the effects of factor interaction. That is, how the effect of factors change when used in combination with others.

The SIMD parallel genetic algorithm is analyzed. Its structure tends to be uniform across many applications and platforms thus identification of factors to investigate requires less subjective judgment. Many SIMD computers are in use and the architecture itself is now re-emerging in the context of special-purpose VLSI plug-ins.

Using experiment design methods tends to require many individual trials over a small number of factors. In this case, more than 2,500 individual instances of the algorithm comprised the single experiment reported. The outcome showed all factors investigated – network topology, migration policy, migration probability, and migration radius—were significant at confidence levels ranging from 90% to 99%. A key question was: Do these factors interact with each other in a simple additive fashion or in a more complex manner? It was found that the interactive effects were sufficiently close to additive that no special precautions by the practitioner are necessary.

\*i2 Technologies, 1603 LBJ Freeway, Suite 780, Dallas TX 75234

†The Center for Intelligent and Knowledge-based Systems, Department of Electrical Engineering & Computer Science, Tulane University, New Orleans, LA 70118. [Dev\\_Prabhu@i2.com](mailto:Dev_Prabhu@i2.com) {buckles, petry}@eecs.tulane.edu

## 2. Background.

**2.1. Sequential GAs.** Genetic algorithms originated from the studies of cellular automata, conducted by John Holland [22]. A GA is a search procedure modeled on the mechanics of natural selection rather than a simulated reasoning process. Domain knowledge is embedded in the abstract representation of a candidate solution termed an organism. Organisms are grouped into sets called populations. Successive populations are called generations. A generation GA creates an initial generation,  $G(0)$ , and for each generation,  $G(t)$ , generates a new one,  $G(t + 1)$ . An abstract view of the algorithm is

```

generate initial population,  $G(0)$ 
evaluate  $G(0)$ 
 $t := 0$ 
repeat
   $t := t + 1$ 
  generate  $G(t)$  using  $G(t - 1)$ 
  evaluate  $G(t)$ 
until solution is found.

```

The operation “evaluate  $G(t)$ ” refers to the assignment of a figure of merit to each of the population’s organisms. In simple GAs, an alternative exists to replacing an entire population at once; that alternative is to replace one organism in the population whenever a new organism is created. This variant is known as a steady state GA.

In most applications, an organism consists of a single chromosome. A chromosome of length  $n$  is a vector of the form  $\langle x_1, x_2, \dots, x_n \rangle$ , where each  $x_i$  is an allele, or gene. The domain of values from which  $x_i$  is chosen is called the alphabet of the problem. Frequently, the alphabet used consists of the binary digits  $\{0, 1\}$ . We can view a specific chromosome as representative of many patterns. Using # as “don’t care,” an example pattern or schema over the binary alphabet is  $\langle \#\#110\# \rangle$ . The chromosomes  $\langle 111100 \rangle$  and  $\langle 101100 \rangle$  are specific instances of this and other schemata; for example,  $\langle \#\#100 \rangle$ . The order of a schema is the number of non-# symbols it contains. Its length is the distance from the first to the last non-# position. Thus, the length of  $\langle \#1\#0\#1 \rangle$  is four, and its order is three.

GAs differ from traditional search techniques in several ways

- First, GAs optimize the trade-off between exploring new points in the search space and exploiting the information discovered thus far. This was proven using an analogy with the  $k$ -armed bandit (an extension of the one-armed bandit) problem.
- Second, GAs have the property of implicit parallelism. Implicit parallelism means that the GA’s effect is equivalent to an extensive search of hyperplanes of the given space, without directly testing all hyperplane values. Each schema denotes a hyperplane.
- Third, GAs are randomized algorithms, in that they use operators whose results are governed by probability. The results for such operations are based on the value of a random number.
- Fourth, GAs operate on several solutions simultaneously, gathering information from current search points to direct subsequent search. Their ability to maintain multiple solutions concurrently makes GAs less susceptible to the problems of local maxima and noise.

As noted, GAs are randomized—but not random—search algorithms. Each organism represents a point (that is, an intersection of hyperplanes) in the search space. Randomization must balance two competing concerns, exploration and exploitation. A solution cannot be tested unless it appears as an organism. Therefore, a reasonable number of solutions must be explored. On the other hand, unlimited exploration would not be efficient search. The strength of highly fit organisms must be exploited and allowed to propagate in the population. Yet, giving too much precedence to such organisms results in premature termination at a local optimum.

We can compare GA recombination operators to controlled breeding among, say, thoroughbred horses. The objective is to combine highly fit organisms to produce a still more fit individual. Both the selection of “parents” and the steps within the recombination operators are randomized. Parent selection dynamics are based on an application-dependent measure of an organism known as the fitness function,  $f_i$  ( $f_i$  is a figure of merit, computed using any domain knowledge that applies). In principle, this is the only point in the algorithm at which domain knowledge is necessary. Organisms are chosen using the fitness value as a guide; organisms having higher fitness values are chosen more often. Selecting organisms based on fitness value is a major factor

in the strength of GAs. The greater the fitness value of an organism, the more likely that the organism will be selected for recombination.

There are two popular approaches for implementing selection. The first, roulette selection, assigns a probability to each organism,  $i$ , computed as the proportion

$$F_i = f_i / \sum_j f_j$$

A parent is then randomly selected, based on this probability. A second method, deterministic sampling, assigns to each organism,  $i$ , a value

$$C_i = RND(F_i \times n) + 1$$

where  $n$  organisms reside in the population ( $RND$  means round to integer). The selection operator then assures that each organism participates as a parent exactly  $C_i$  times.

Parents participate in the later recombination operations. Alleles from the parents are mixed via an operator called a crossover rule, of which many exist. Simple one-point crossover of chromosomes from two parents at a random point,  $j$ , is illustrated by

$$\langle x_1 \ x_2 \ \cdots \ x_j \ x_{j+1} \ x_{j+2} \ \cdots \ x_n \rangle \quad (2.1)$$

$$+ \quad (2.2)$$

$$\langle y_1 \ y_2 \ \cdots \ y_j \ y_{j+1} \ y_{j+2} \ \cdots \ y_n \rangle \quad (2.3)$$

$$= \quad (2.4)$$

$$\langle x_1 \ x_2 \ \cdots \ y_j \ y_{j+1} \ y_{j+2} \ \cdots \ y_n \rangle \quad (2.5)$$

where the result is a chromosome of the offspring and is placed in the next generation.

A mutation is the random change of an allele from one alphabet value to another. For a problem over the binary alphabet, the original allele is exchanged for its complement. The mutation operator offers the opportunity for new genetic material to be introduced into a population. From the theoretical perspective, it assures that—given any population—the entire search space is connected. The new genetic material does not originate from the parents and is not introduced into the child by crossover. Rather, it occurs after crossover a small percentage of the time.

Several stopping criteria exist for the algorithm. The algorithm may be halted when all organisms in a generation are identical, when  $f_i = f_j$  for all  $i$  and  $j$ , or when  $|f_i - f_j| < TOL$  for some small value  $TOL$  and all  $i$  and  $j$ . An alternative criterion would halt after a fixed number of evaluations and take the best solution found.

**2.2. Parallel GAs.** Parallel versions of GAs have, more often than not, different semantics from the canonical serial algorithm [6, 45]. In other words, the behavior of the algorithm in terms of the computational actions taken as well as the results generated for any given set of inputs can be expected to differ for the two versions. In contrast, for most other algorithms, parallelization usually implies a parallel version of the algorithm which maintains the semantics of the serial version. Not surprisingly, early parallel GA implementations (e.g. [34]) focused mainly on achieving computational speedup. There have been a variety of applications of parallel GAs including topics such as fuzzy logic controller design [1], VLSI routing [25], financial market computations [32], SAT methods [15], land use modeling [49], and many others [2].

The changed semantics of a parallel GA is primarily the result of the way in which the population members are distributed among subpopulations as well as the nature and the extent of interactions among such subpopulations. A serial GA which uses a single centralized population and global access to all its members (as evidenced by panmictic selection) simply constitutes one end of a spectrum of possibilities [8]. At the other end of that spectrum lies a fine-grained parallel GA with completely decentralized population (resulting in singleton subpopulations) and a local neighborhood-based selection. Most other parallel GAs can be seen as being somewhere in between these two cases. They employ a varying extent of decentralization of the global population and access among the resulting subpopulations during selection. The subpopulations resulting from the distribution of the global population members can be called *demes*, a very evocative term from the field of population biology. Parallel GA literature also refers to them as *islands* [48, 27].

The interaction among the demes can take two forms: *migration* and *overlapping selection* [7]. Migration refers to the process of periodic import of population members from neighboring demes into the local subpopulation. In the artificial setting of parallel GAs, this involves a sequence of export-import-replace actions. First, every deme must decide on candidates for export or emigration from the local subpopulation, which in turn can be imported by the neighboring demes. Next, a deme can decide to import or immigrate some of such candidate emigrants available in the neighboring demes. The frequency of import is determined by the *migration probability*. Since most parallel GA models enforce fixed size populations, a necessary third step would be to incorporate the newly imported members into the local subpopulation by replacing local members.

A different kind of interaction among the demes is based on the notion of selection using effectively overlapping subpopulations. In this scheme, during the selection and reproduction stage of GAs, some of the neighboring subpopulations are also taken into consideration along with the local subpopulation. Thus, using  $k$  neighboring demes and a subpopulation size of  $n$  at each deme, selection and reproduction step results in sampling  $n$  members from the pool of size  $(k + 1) \times n$ . Depending on such overlap, the effective subpopulations employed over all the demes constitute either a partition or a covering set of the global population. Obviously, a meaningful migration operation in such a model can be designed only by considering the demes which are not in the same selection pool.

**2.3. Related Research.** Here we focus on parallel GAs that support the deme concept. It was recognized early that a variety of choices existed with respect to the spatial distribution of population members [41, 5]. At one end of the spectrum are so-called coarse-grained parallel GAs [3]. In this case, the global population is divided into a few distinct non-overlapping subpopulations. Communication among them is accomplished by periodic migration [12, 13]. Selection within a subpopulation is an issue but there are no inter-deme selection issues. If a parallel computer is applied to coarse-grained GA simulation, a MIMD architecture is most appropriate [43].

Rigorous experimentation with coarse-grained GAs was first performed by Tanese [44]. She examined the frequency and subpopulation fraction for migration. It was discovered that moderate values of each are the most effective. She also noted something else that was peculiar: Even in the absence of any inter-deme communication, the parallel GA with small subpopulations performed better than the serial GA. Forrest *et al.*, in follow-up work [17], determined that this was an artifact of the problem to which the GA was applied (a subset of Walsh functions). The coarse-grained approach allows considerable freedom such as varying control parameters, e.g., crossover rate, between subpopulations [44], varying the representation from one subpopulation to the next [37], and even dynamically varying the subpopulation sizes and number of subpopulations [46, 20].

In contrast, a fine-grained parallel GA is, strictly speaking, one in which the number of demes is the same as the population size. That is, each deme consists of one individual. A classic example is one reported by Manderick *et al.* [26]. In this class of GAs the principal issue is selection policy (radius, for example) [14]. A SIMD architecture is the more appropriate parallel computer for fine-grained GA simulations. Some other terms that have been used for fine-grained GAs include cellular GAs [47] and diffusion GAs [33]. The fine-grained approach has also been applied to evolution strategies to study different selection schemes [18]. Cellular programming of cellular automata is an approach that is also related to fine-grained parallel GAs [40, 9].

Mühlenbein initially developed a fine-grained GA [30] on a double-ring grid that incorporated hill-climbing. Later, the system was extended [31] in a manner that incorporated non-singleton subpopulations with both inter-deme selection and migration issues and so was not strictly fine-grained. The algorithm we have used is also not strictly fine-grained, but tends in that direction [35]. Small subpopulations were isolated with respect to selection but connected with respect to migration. This was in accordance with the controlled variables we wished to study. Otherwise, the algorithm as illustrated in Figure 2.1 has all the recognizable elements of a serial GA.

**3. Logical Architecture.** This experiment focuses primarily on the interactions of subpopulations. Such interactions are constrained by the connectivity among the subpopulations and the consequent neighborhood structure within the logical architecture of the parallel GA. The physical architecture does not prohibit any specific logical architecture. Suppose that  $n$  nodes are given. Any graph that weakly connects them induces a *topology*. The topologies considered in this study further assume that the graph is not directed. Specifically, the four topologies considered are shown in Figure 3.1. Choice of a topology results in a *neighborhood* for each node. A neighborhood can be compact or not. A compact neighborhood of size  $r$  consists of a reference node together with all other subpopulations reachable by traversing at most  $r$  links of the topology. A noncompact

```

Set the number of subpopulations;
Set the topology and migration radius;
For all nodes synchronously Do:
{
  Randomly generate initial subpopulations;
  Evaluate all members of initial subpopulations;
  While termination conditions not met, Do:
  {
    /* Migration stage */
    Export: for every node copy a candidate to export;
    Choose a subset of nodes based on migration neighborhood;
    For these nodes synchronously Do:
    {
      Import: Pick zero or one from the import candidates based on
              migration probability;
      Replace: Replace a local member by candidate chosen;
    }
    endFor

    /* Reproduction stage */
    Selection: For each node, assign fitness-based target
              sampling rates; /* ideal value for re-use as parent */
    Sampling: Sample members based on target sampling rates;

    /* Transformation stage */
    Crossover: Stochastically recombine pairs generating offspring pair;
    Mutation: Stochastically make small changes in offspings;

    /* Evaluation and iteration stage */
    Evaluate each new individual;
    Replace parent subpopulation by offsprings; /* generational GA */
  }
  endWhile
}
endFor

```

FIG. 2.1. *The parallel genetic algorithm*

neighborhood consists of any other strongly connected subgraph of the topology. Figure 3.2 shows an example of compact neighborhoods for  $r = 1$  and  $r = 2$  for a node in a 4-neighbor torus topology.

Selection using overlapping subpopulations—a form of subpopulation interaction—is based on the notion of compact neighborhoods. We define the *selection neighborhood* of size  $r_s$ , termed *selection radius*, for a node to be the corresponding compact neighborhood of that node. Let  $K_S$  denote the set of subpopulations in the neighborhood. By extending the chances of reproduction in the local node to members of all the subpopulations in the selection neighborhood, overlapping subpopulations can be effectively simulated based on the selection neighborhood. We consider only the cases for which  $r_s = 0$ , that is, the selection neighborhood is the local subpopulation.

Subpopulation interaction by migration provides access to members of subpopulations that lie outside the selection neighborhood. Given a selection radius of  $r_s$ , we define the *migration neighborhood* of size  $r_m$  to be the noncompact neighborhood resulting from deleting the selection neighborhood from the compact neighborhood of size  $(r_s + r_m)$  where  $r_m$  is termed *migration radius*. In other words, if we denote  $K_{S+M}$  to be the set of

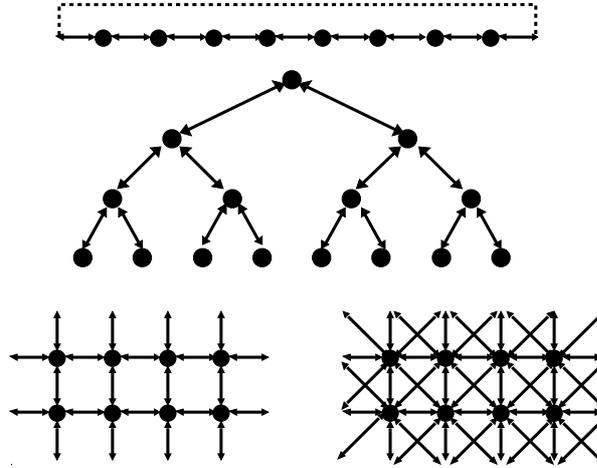


FIG. 3.1. Subpopulations in ring, tree, 4- and 8-neighbor torus topologies. Open-ended arrows denote wrap-around connectivity.

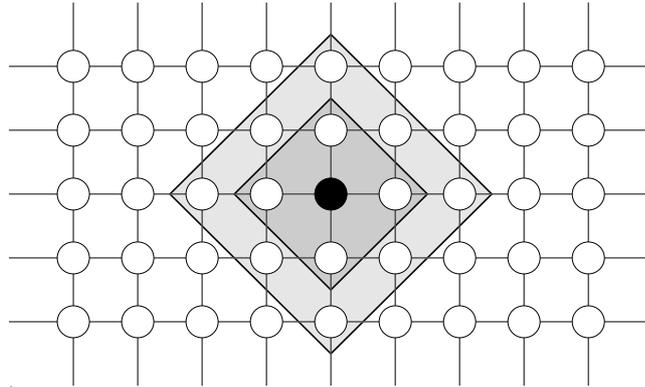


FIG. 3.2. Compact neighborhoods for a node of a four-torus topology. The nodes in the inner shaded area are included for  $r = 1$ . The nodes in both the shaded areas are included for  $r = 2$ .

subpopulations for the size  $(r_s + r_m)$ , then the set of subpopulations available for the migration operation is defined by  $(K_{S+M} - K_S)$ . Figure 3.3 illustrates a migration neighborhood for  $r_s = 1$  and  $r_m = 1$ . For a zero-valued selection radius, the migration neighborhood reduces to the compact neighborhood of size  $r_m$ , sans the reference node.

**4. Experimental Design.** Many empirical studies reported in the computer literature compare one method with another or the effect of one factor. This is in contrast to the natural sciences in which it is common to simultaneously study the effects of a number of factors and their interactions. The study here is closer in spirit to the principles of statistical experiment design as first used in agricultural, and then manufacturing, settings. At least one previous study [39] of GAs has adopted a similar strategy.

Specifically, we employed an  $(m_1 \times m_2 \times m_3 \times m_4)$  – factorial design with complete randomization of trials. That is to say, there were four *factors* (independent variables) with  $m_1, m_2, m_3$ , and  $m_4$  levels, respectively. A *level* is a quantitative or qualitative value at which a factor is tested. An *experiment* consists of a set of trials. A *trial* is a test involving each factor set at one of its levels. (In this experiment, a trial consists of one complete run.) Each trial corresponds to a single measurement of a *response variable* (dependent variable). Trials at the same factor levels are repeated several times with different initial conditions in order to obtain meaningful statistical properties.

Trials in an experiment must be conducted using a specific problem. An appropriate problem is one that is both generic and which provides insight into the behavior of the algorithm. Holland’s revised version of Royal Road functions meets these criteria [24]. The Royal Road function chosen is particularly large and difficult to optimize. It is sufficiently difficult to exhibit meaningfully different behavior under different experimental

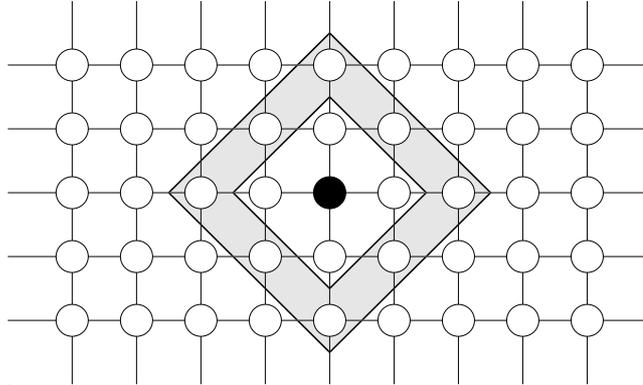


FIG. 3.3. Migration neighborhood for a node in four-torus topology, given  $r_s = 1$ . The nodes in the shaded area are included by a migration radius of  $r_m = 1$ .

conditions as is needed in studies such as this. The Royal Road functions were designed to reward the discovery of lowest level building blocks of the optimal solution as well as the successful recombination of such blocks into higher level blocks in a hierarchical fashion. Royal Road functions have been widely used in a number of areas, for example, crossover variations [42], dual GAs [50] and applications such as learning of neural networks [23].

Here we briefly describe the function adopting the symbols and terminology used by Holland [28] and Jones [24]. For this problem, populations are formed from individuals over the binary alphabet. The structure of the solutions in the population is uniformly specified by the triple  $\langle b, g, k \rangle$ . Each genome is composed of  $2^k$  basic blocks of length  $(b + g)$ . Thus, each individual consisted of 240 genes using the parameter values shown in Table 4.1. Within each block, allele values at pre-specified  $b$  positions (block bits) are relevant for fitness computation and the allele values of the remaining  $g$  positions (gap bits) do not affect the fitness. Such basic blocks are called level-0 blocks. A level-0 block is considered complete when all of its block bits take the value 1. Even-odd pairs of level-0 blocks comprise level-1 blocks. Level- $i$  blocks, for  $1 \leq i \leq k$ , are comprised of even-odd pairs of level- $(i-1)$  blocks. Further, any level- $i$  block is considered complete if the two adjacent blocks forming its constituent level- $(i-1)$  even-odd pair are both complete.

The computation of the Royal Road function used itself can be described in two parts, typically referred to as the PART and the BONUS computations. PART computes a value with respect to level-0 blocks using two parameters,  $m^*$  and  $v$ . Let  $j$  stand for the number of 1s present in the  $b$  block bits for a given level-0 block in the genome. Now, the PART contribution for that block is given by  $(j \times v)$ , if  $j \leq m^*$ , and by  $((m^* - j) \times -v)$ , if  $m^* < j < b$ . The contribution is zero (0) if  $j = b$ . The full PART value is computed by summation over all the level-0 blocks. The intentional penalties for near-optimal blocks at the lowest level make this problem harder for hill-climbing algorithms.

The BONUS computation rewards the presence of complete blocks at each level  $i$ ,  $0 \leq i \leq k$ , and is specified by the parameters  $u^*$  and  $u$ . Recall that only the presence of a corresponding even-odd pair of complete blocks of level- $(i-1)$  can generate a complete block at level- $i$ . Let  $j$  stand for the number complete blocks present in the genome for a given level- $i$ . The BONUS contribution for level- $i$  is zero, if  $j = 0$  and is given by  $(u^* + (j - 1) \times u)$  for  $j > 0$ . The complete BONUS value for the genome is computed by summation over all the levels. The fitness function itself is again a sum of PART and BONUS values. The parameter values used in this study are shown in Table 4.1 and they are similar to the values suggested by Holland.

**4.1. Factors.** The four factors studied are given in Table 4.2. Each was tested at four levels, leading to a  $4 \times 4 \times 4 \times 4$  factorial experiment. Two factors—migration radius and migration probability—are quantitative. Two—topology and migration operator—are qualitative. Thus the experiment consisted of 256 combinations of different levels of factors, or “cells”. Each such cell was sampled ten times resulting in a total of 2560 runs of the parallel GA using the same set of ten random seeds for each cell. The sequence of runs was determined in a fully randomized fashion. The parameters held constant over the experiment are listed in Table 4.3. The values in Table 4.3 were chosen following an extensive study of the SIMD algorithm literature. They appear to be robust and generally accepted in applications. Each subpopulation exported one individual (by copy) during each generation. When a subpopulation chose to import, it imported one individual from the set of candidates

TABLE 4.1  
Parameters for the Royal Road function.

| Parameters | Value |
|------------|-------|
| $k$        | 4     |
| $b$        | 8     |
| $g$        | 7     |
| $m^*$      | 4     |
| $v$        | 0.02  |
| $u^*$      | 1.0   |
| $u$        | 0.3   |

TABLE 4.2  
Experimental Factors

| Factor                | Symbol | Levels  |
|-----------------------|--------|---|
| Topology              | $G$    | { $G_{.0}$ = Ring, $G_{.1}$ = Tree,<br>$G_{.2}$ = 4-Torus, $G_{.3}$ = 8-Torus }   |
| Migration Radius      | $R$    | { $R_{.0}$ = 1, $R_{.1}$ = 3, $R_{.2}$ = 5, $R_{.3}$ = 7 }  |
| Migration Operator    | $O$    | { $O_{.0}$ = Import-random & Replace-random,<br>$O_{.1}$ = Import-random & Replace-worst,<br>$O_{.2}$ = Import-best & Replace-random,<br>$O_{.3}$ = Import-best & Replace-worst } |
| Migration Probability | $P$    | { $P_{.0}$ = 0.0, $P_{.1}$ = 0.33, $P_{.2}$ = 0.66, $P_{.3}$ = 1.0 }  |

available. The experiments were performed on the MasPar MP-216 at NASA/GSFC using MPGA [36], a massively parallel GA package developed by the authors.

**4.2. Response Variables.** The effect of the factors was tested using response variables corresponding to population diversity, schemata propagation, and search efficiency. More than one method was employed [35], but here we restrict the discussion to one representative method for each.

An objective in a GA, parallel or serial, is to obtain a balance between population diversity and the speed of convergence, *i. e.*, exploration and exploitation. Therefore studying the effect of the factors on the population diversity is important. To measure diversity we use a metric similar in spirit to the one described by Collins [13, pages 118–120].

$$\delta = \frac{1}{l} \times \sum_{i=1}^l \left( 1 - 2 \times \left| 0.5 - \frac{\sum_{j=1}^m \sum_{k=1}^n \text{Bit}(i, k, j)}{m \times n} \right| \right) \quad (4.1)$$

where,  $l$  is the length of the chromosome in bits,  $m$  is the number of subpopulations, and  $n$  is the number of chromosomes in each subpopulation.  $\text{Bit}(i, k, j)$  denotes the value of the  $i$ th bit of the  $k$ th member in the  $j$ th subpopulation.  $\delta$  is in the range  $[0, 1]$  where zero (0) is the response for a global population consisting of one individual replicated and one (1) is the response when the global population is maximally diverse, *i. e.*, at each position all alleles are uniformly distributed.

Schemata propagation is used here to mean the extent to which a fit schema is present among all the subpopulations. A priori choice of a single schema to be monitored is not practical because there are a number of search paths from different schemata that lead to an optimum. Taking advantage of the problem symmetry, eight complementary schemata were chosen for monitoring. These schemata are shown in Figure 4.1. At selected generations the subpopulations were examined for presence of the schemata. In the results of Section 5 the final measurement is used.

The response value was not simply the fraction of subpopulations in which a schema was found. It was computed as the number of subpopulations containing instance(s) of the schema less the subset of those demes

TABLE 4.3  
GA parameters used in the experiment.

| Parameters                    | Value                         |
|-------------------------------|-------------------------------|
| Number of subpopulations      | 16,384                        |
| Subpopulation size            | 10                            |
| Number of maximum generations | 100                           |
| Export operator               | Export-current-best           |
| Selection radius              | 0                             |
| Selection operator            | Ranking                       |
| Elitism                       | No                            |
| Sampling operator             | Stochastic universal sampling |
| Mutation operator             | Bit-mutation                  |
| Mutation probability          | 0.005 per bit                 |
| Crossover operator            | Two-point crossover           |
| Crossover probability         | 0.7                           |

having no neighbors (with respect to the topology) also containing instance(s). That is, only those subpopulations in which the schema could plausibly have occurred as a result of migration are counted. The maximum for the eight candidate schemata was chosen as the basis for the response. This accounts for the various search paths the GA might traverse. The response variable was normalized to be the fraction of the number of applicable subpopulations meeting the propagation criteria.

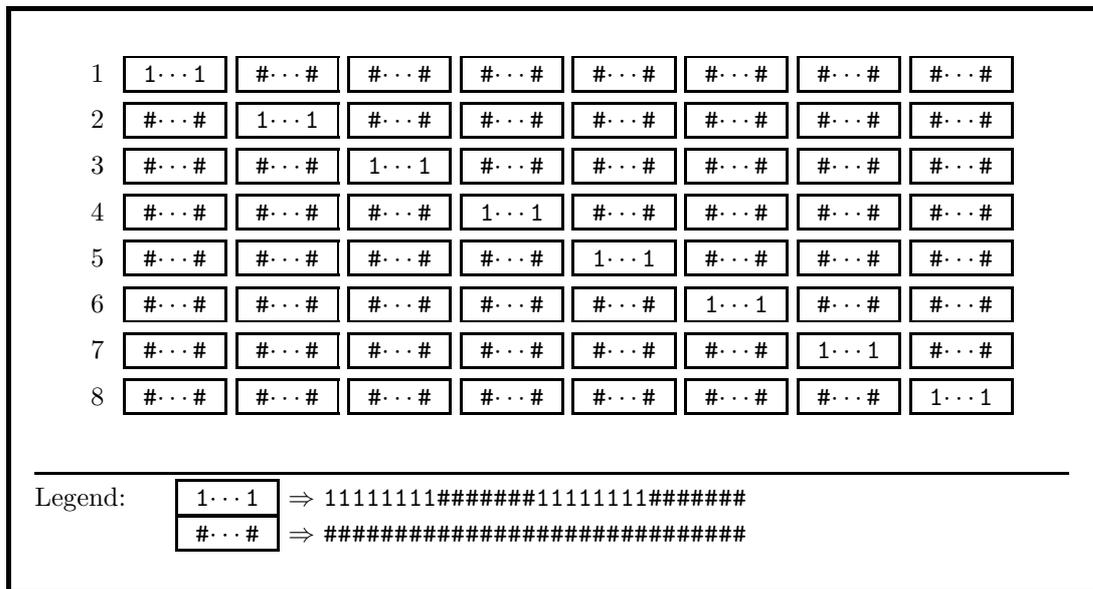


FIG. 4.1. The list of schemata monitored.

Finally, search efficiency is measured in terms of the time taken to reach the optimum. In this case, the number of generations taken to find the globally optimal solution is recorded. If a particular trial failed to locate the optimum, the preset value of maximum number of generations was used. Typically, the value for efficiency tends to be domain-specific. However, it is hoped that the premise underlying the design of the Royal Road functions—*building-block hypothesis*—renders the results a significance sufficiently independent of specific applications.

**5. Results and Analysis.** An important aspect of this paper is the experimentation and analysis of variance methods employed. It is more common to see less formal experiments having a different objective—performance prediction. The formal experimental design approach used here discovers which factors affect

performance. A factor may affect performance by itself, in combination with other factors, or both. The most important result here, given that each factor investigated affected performance as might be expected, is that there were no unusual effects that occurred when factors were used in combination. That is, the effects in combination are a linear combination of the effects of each single factor.

**5.1. ANOVA.** An illustrative ANOVA is shown in Table 5.1. A complete table for each response variable is in the appendix. The source column lists the factor (e.g.,  $R$  or migration radius) or factor combination (e.g.,  $O \times P$  or migration operator with migration probability) that was tested for its effects. The last column (Computed  $F$ ) is a statistic that is compared to standard tables to extract the degree of confidence that the source factor or source combination has a significant influence on the response. Further description of the ANOVA tables and the columns can be found in the appendix.

TABLE 5.1  
*Abbreviated ANOVA for global allelic diversity*

| # | Source                         | DF | Sum Sq  | Mean Sq | Computed $F$ |
|---|--------------------------------|----|---------|---------|--------------|
| 1 | $R$                            | 3  | 3.8493  | 1.2831  | 40437.7601   |
| 2 | $P$                            | 3  | 81.5102 | 27.1701 | 856282.9984  |
| 3 | $R \times O$                   | 9  | 0.4826  | 0.0536  | 1689.9835    |
| 4 | $O \times P$                   | 9  | 7.1838  | 0.7982  | 25155.7570   |
| 5 | $G \times R \times O$          | 27 | 0.1644  | 0.0061  | 191.8747     |
| 6 | $G \times O \times P$          | 27 | 1.0641  | 0.0394  | 1242.1203    |
| 7 | $G \times R \times O \times P$ | 81 | 0.3613  | 0.0045  | 140.5775     |

Since a large computed  $F$ -statistic value relative to the corresponding critical value indicates that the effect under scrutiny is significant under normal circumstances, the ANOVA tables shown would lead one to conclude that all the factors and their interactions have significant effect on the response variables. This is, in fact, conclusive with respect to the single factor rows in Table 5.1 and the tables in the appendix. However, it is not conclusive with respect to the factor interaction effects. Their appearance of significance may be a carry-over effect of the individual factor effects. The standard recourse in such instances is to scrutinize in greater detail the effects. Generally, a graphical approach is used and that approach is taken in the following sections. For completeness, we examine the individual factors this way as well as the factor interactions.

**5.2. Effects of individual factors.** Note that in the ANOVA analysis, Tables A.1, A.2, and A.3, the computed  $F$ -statistic values for migration probability factor are the greatest. Migration probability, in fact, clearly affects all three responses as is evident from Figure 5.1. In the single factor graphs, the feature one looks for in order to confirm visually that a factor affects performance is a curve with a slope other than zero. The response values ( $Y$ -axis) for these and succeeding plots are described in Section 4.2. To summarize, the  $Y$ -axis of the top graph of Figures 5.1-5.4 is the value from Eq. 4.1. The  $Y$ -axis of the middle graph of the figures is the fraction of subpopulations in which a key schema occurs (plausibly) by means of migration. Finally, the  $Y$ -axis of the bottom graph is the number of generations required for convergence. For single-factor plots, each data point is an average over all possible combinations of other factors, *i. e.*, each point is an average of 640 values. The error bars depict the standard deviation.

Figure 5.1 emphatically demonstrates that subpopulations when interconnected behave differently than when isolated, *i. e.*,  $P = 0$ . Further, up to a point, the level of intercommunication, as reflected by the magnitude of migration probability, makes a difference. In the absence of migration, *i. e.*, zero probability, there is little exploitation (*i. e.*, maximal diversity), schema proliferation only by chance rediscovery, and poor search efficiency. In contrast, moderate to high migration probabilities result in significantly different behavior with respect to all response variables. As is seen in the following subsections, migration probability also exhibits significant interaction with other factors.

In Figure 5.2, it can be seen that the migration operator also affects all three responses. The effects, however, are less dramatic compared to the effect of migration probability. Recall that the migration operator components include an import policy and a replacement policy. Figure 5.2 suggests that the former is more important than the latter. In each plot the first data point is similar to the second and the third data point is similar to the last. See again Table 4.2 to correlate the data points with migration policies.

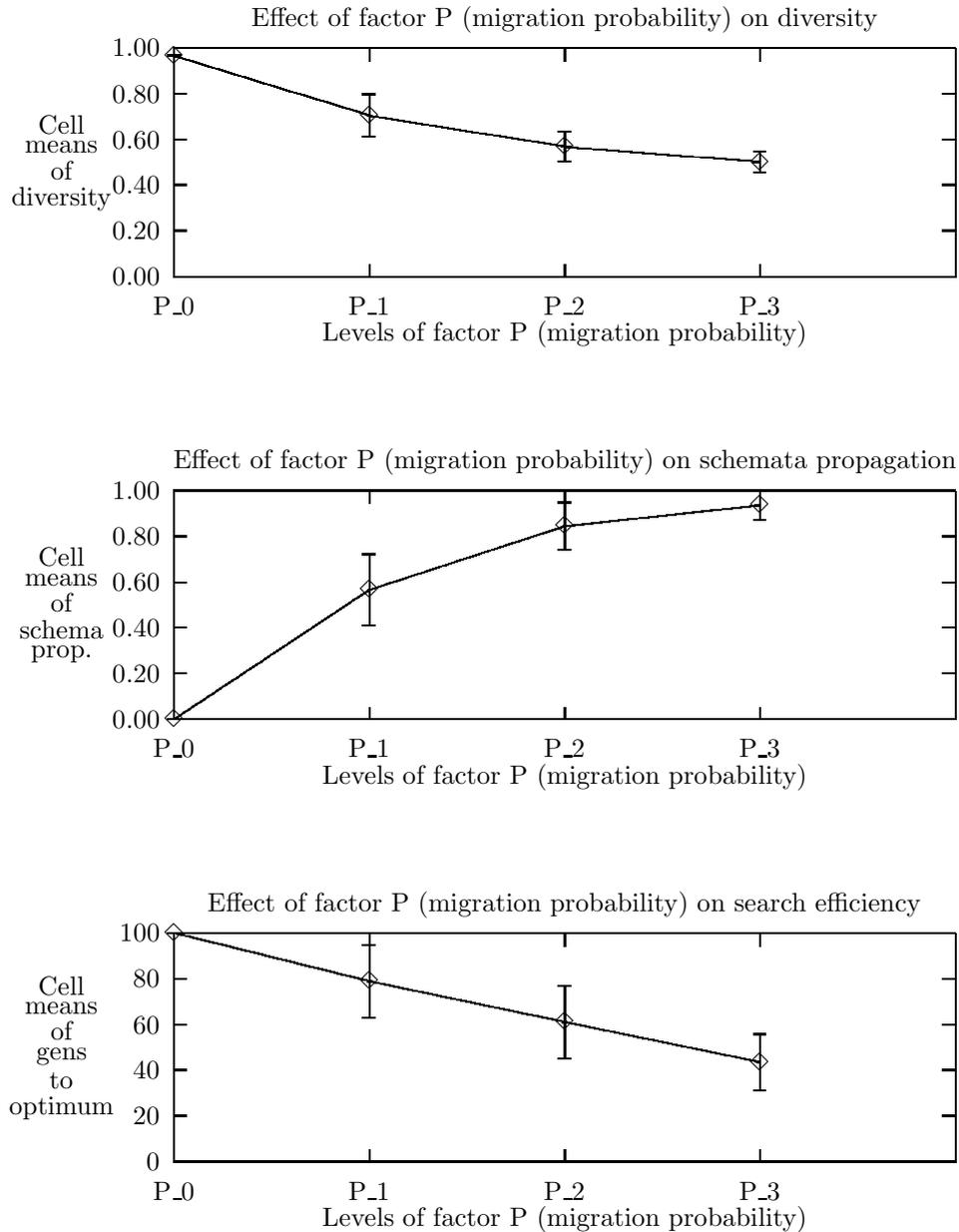


FIG. 5.1. *Effect of factor P (Migration Probability) on the response variables*

Figures 5.3 and 5.4 indicate that the independent effects of topology and migration radius are minimal. The changes in responses between the first and the second data points of each plot might best be explained in terms of changes in the degree of connectivity. The similarity among the remaining data points suggest that further increases in the degree of connectivity have little effect.

**5.3. Effects of multiple factor interactions.** One method for examining in more detail the higher order interactions is to visually compare different components of the effect under scrutiny, using two factors at a time. The cell means of the response variable are plotted against one of the factors, for each level of the second factor. All the cell means are averaged over all the combinations of levels of the remaining factors.

To illustrate the approach, the effects of the two factor interactions—migration radius vs. migration operator and migration operator vs. migration probability—on the response variable measuring diversity are shown in Figures 5.5 and 5.6 respectively. The null hypothesis here is: The effects of factor level combinations are an

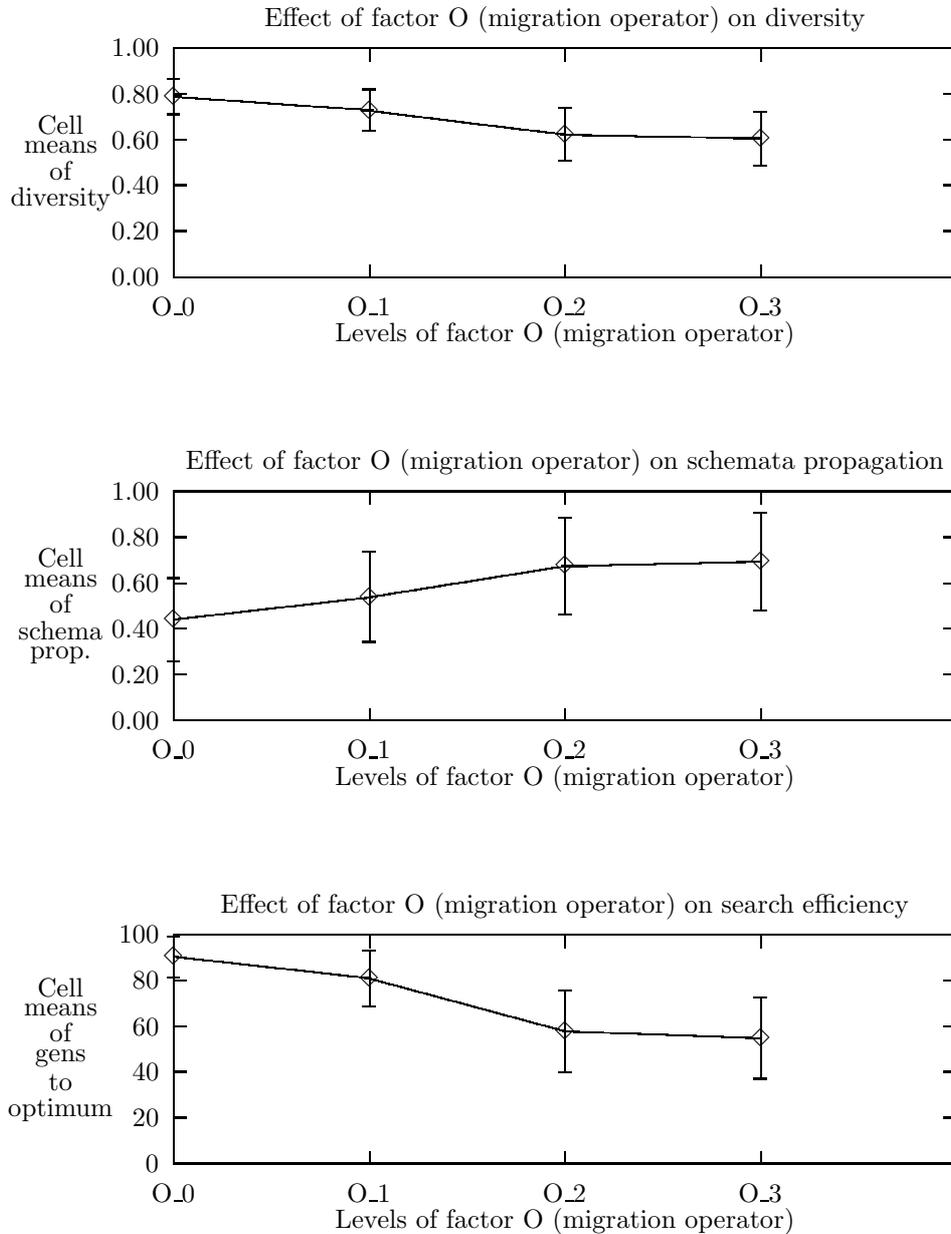


FIG. 5.2. *Effect of factor O (Migration Operator) on the response variables*

additive (linear) combination of the individual factor levels. The visual feature one looks for to “not reject the null hypothesis” is roughly parallel lines. The four plots in Figure 5.5 are roughly parallel to each other, thus the interaction effect is not significant with respect to diversity. Figure 5.6 shows an example that might suggest some factor interaction but the lines are parallel statistically speaking.

When we consider any three-factor interaction, one should examine all the plots that can be generated by choosing two factors at a time for each level of the third factor. For any response variable there are twelve graphs for each of the four possible three-factor interactions. If most of those graphs show parallel plots, we can conclude the effect of the interaction is insignificant. A similar procedure can be carried out to determine the significance of the four-factor interaction.

The complete analysis for three response variables of a  $4 \times 4 \times 4 \times 4$  experiment uses more than 450 such graphs. Here we selectively present those which best illuminate the behavior. More details and a complete set of graphs can be found in [35].

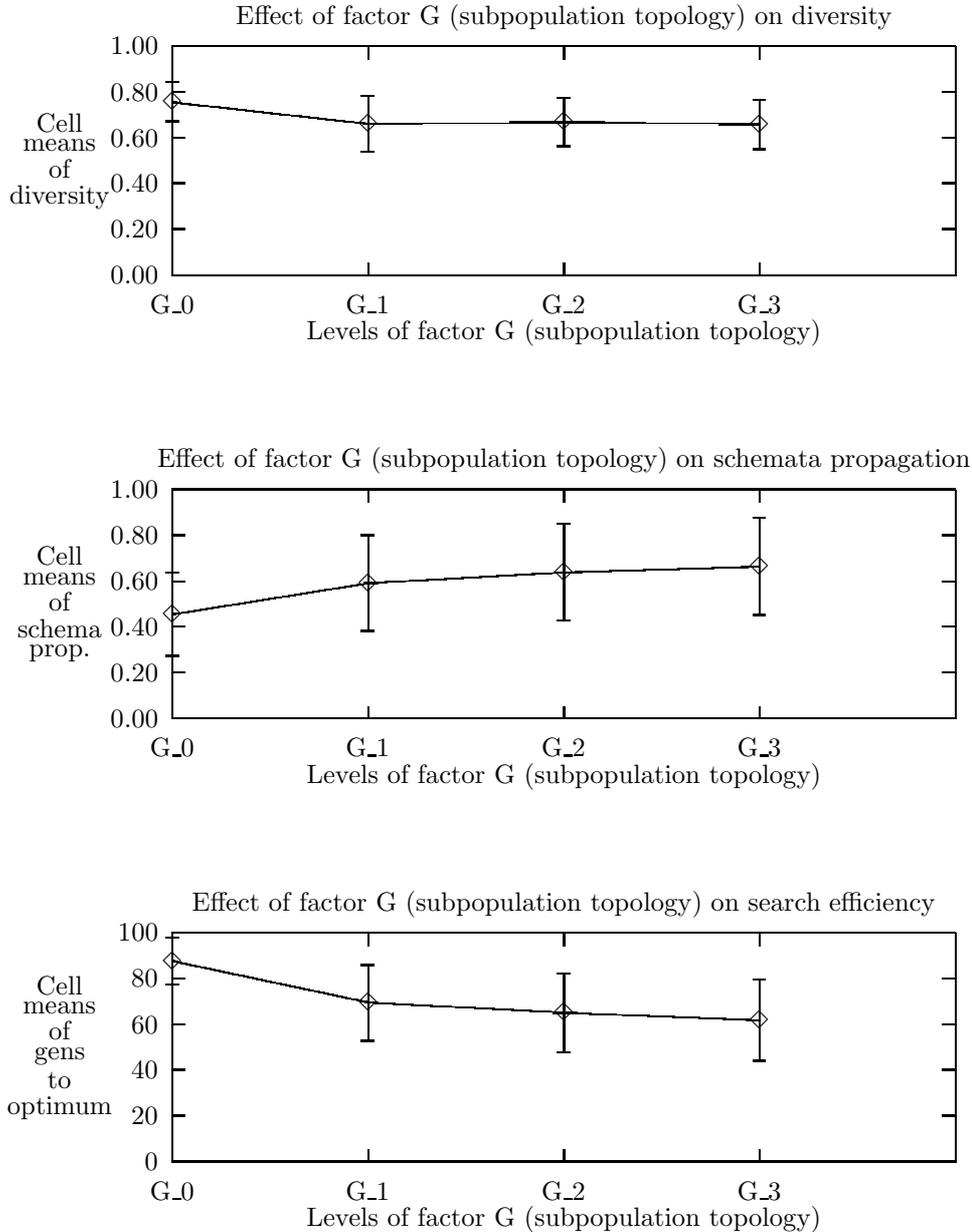


FIG. 5.3. *Effect of factor G (Subpopulation Topology) on the response variables*

Since the migration probability and migration operator have the greatest single factor effects, as can be expected, they exhibit the most pronounced second-order effects. The second order effect of  $O \times P$  on diversity is shown in Figure 5.6. Similar behavior can be observed with respect to the other response variables. In Figure 5.6, there is evidence explaining the interaction of  $O \times P$ . Namely, the import policy impacts the migration effect to a greater extent than the replacement policy. Also, reinforcing our expectations, a migration probability of zero produces the most distinctively different results.

Given that  $O \times P$  is the most significant second order interaction, we show first an example of a three-factor interaction not involving both simultaneously. Figure 5.7 depicts some of the effects of the interaction among migration radius, migration operator, and topology, *i. e.*,  $R \times O \times G$ , on the response variable measuring schemata propagation. It compares the first factor against the second while keeping the third factor constant at some level. In this figure, only the first graph, which corresponds to ring topology, indicates some significance.

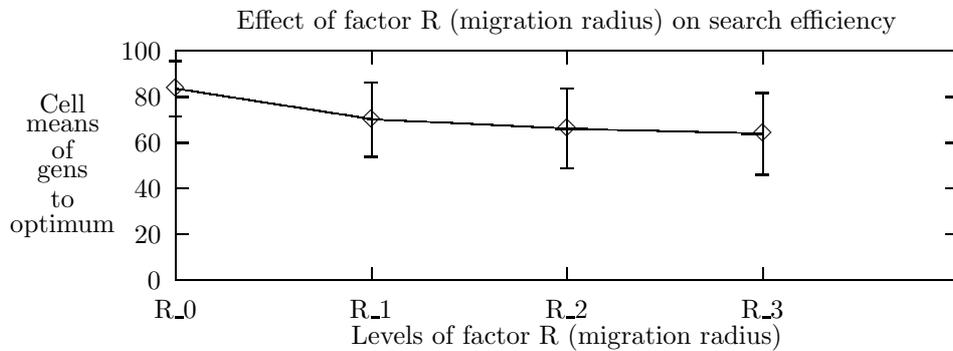
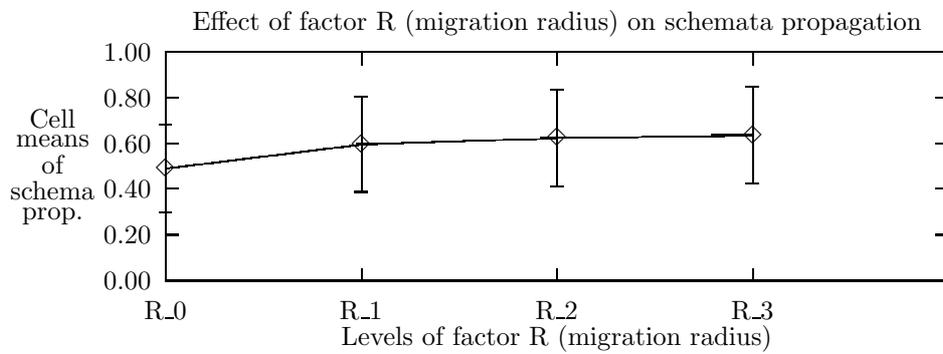
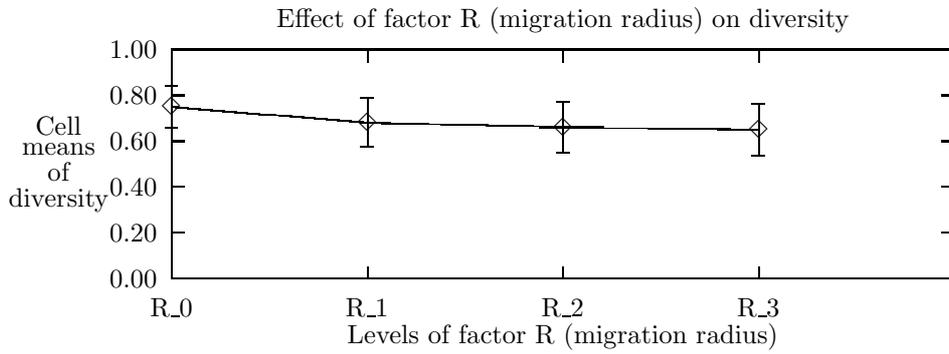


FIG. 5.4. *Effect of factor R (Migration Radius) on the response variables*

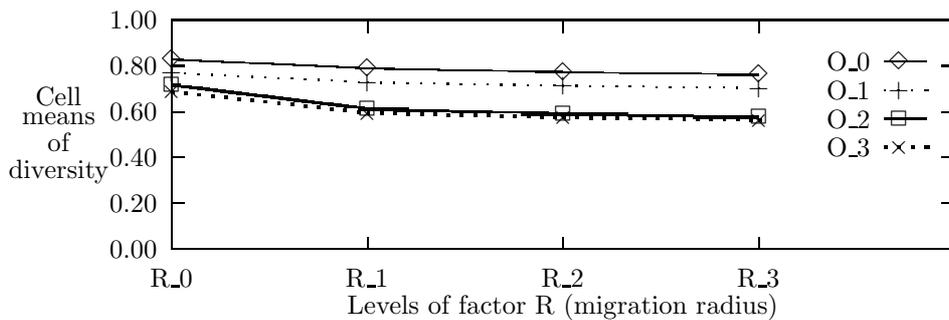
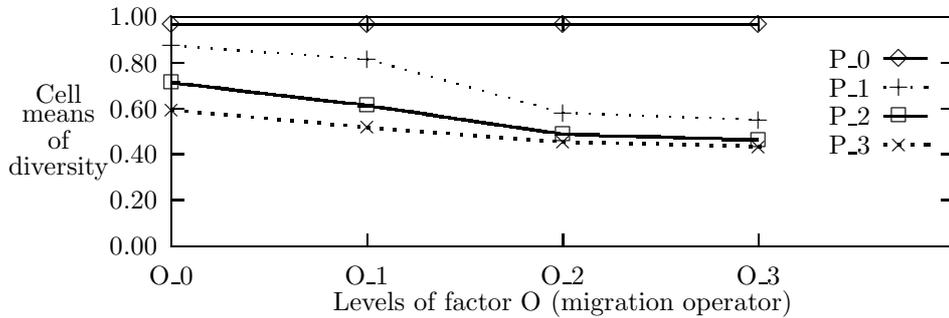


FIG. 5.5. *Effect of RxO interaction on diversity*

FIG. 5.6. *Effect of  $O \times P$  interaction on diversity*

This is probably a reflection of the effect migration radius has in a low-connectivity architecture. The plots in terms of other response variables indicate essentially the same behavior.

Contrast Figure 5.7 with Figure 5.8 in which the effects of the interaction among migration radius, migration operator, and migration probability on schemata propagation are shown. While the  $R \times O \times P$  interaction effect is not highly pronounced in the figure, it does assert itself in the second plot. This again is further corroboration of the importance of the import policy component of the migration operator.

The computed  $F$ -statistic for the fourth order interaction is low in a relative sense in each of the ANOVA Tables A.1, A.2, and A.3 shown in the appendix. This leads one to suspect that the interaction effect is minimal. Figure 5.9, which illustrates the fourth order interaction partially, supports this observation. The plots are in terms of the search efficiency response but the graphs of other response variables lead one to the same conclusion.

**5.4. Global Behavior.** Recall that in the experiments, the number of cells was 256 ( $4 \times 4 \times 4 \times 4$ ). Since we took 10 samples for each cell (using the same set of 10 random seeds across all the cells) a total of 2,560 runs were made. Of the 2,560 runs, the algorithm found the maximum royal road (RR) level of 4 in 1,316 runs, reached RR level 3 in 1,834 runs, reached RR level 2 in 1,920 runs. It reached the first level in all runs. (While we are speaking here of global behavior, let us remind the reader the 640 runs that it failed to climb above level 1 are the same runs for which the migration probability was zero, i. e., the subpopulations were isolated. Table 5.2 summarizes the global behavior. We suggest that the measure “Evaluations (Avg.)”

TABLE 5.2  
*Summary across experiments.*

| RR Level | Reached in ... | That is ...  | Fuction Evaluations (Avg.) | Generations (Avg.) |
|----------|----------------|--------------|----------------------------|--------------------|
| 1        | 2,560 runs     | 100% of runs | 163,840                    | 1.00               |
| 2        | 1,920 runs     | 75% of runs  | 2,723,840                  | 16.63              |
| 3        | 1,834 runs     | 72% of runs  | 6,102,459                  | 37.25              |
| 4        | 1,316 runs     | 51% of runs  | 7,809,790                  | 47.67              |

(the number of function evaluations required to reach a specific level) is not as informative as the number of generations given that 16,384 evaluations are done in parallel. That is 16,384 evaluations occur between points at which the level noted.

In general, suppose someone were to budget or use the same computational effort using a non-parallel GA. An example would be a GA with population size of 100 running for 81,920 generations. Would that setup reach RR level 4 much faster than the 8,192,000 evaluations used in the MPGA of Table 5.2? That is, would the algorithmic performance of such a setup, as measured by machine cycles needed to find the optimal fitness vlaue, be much better? We think that is extremely likely to be true. For example, you might reach the same fitness value by 5,000 generations. However, the wall-clock time for 5,000 generations will be prohibitive, especially for complex fitness functions. On the other hand, the actual number of fitness function evaluations is not that important for massively parallel GAs because the required wall-clock time is small.

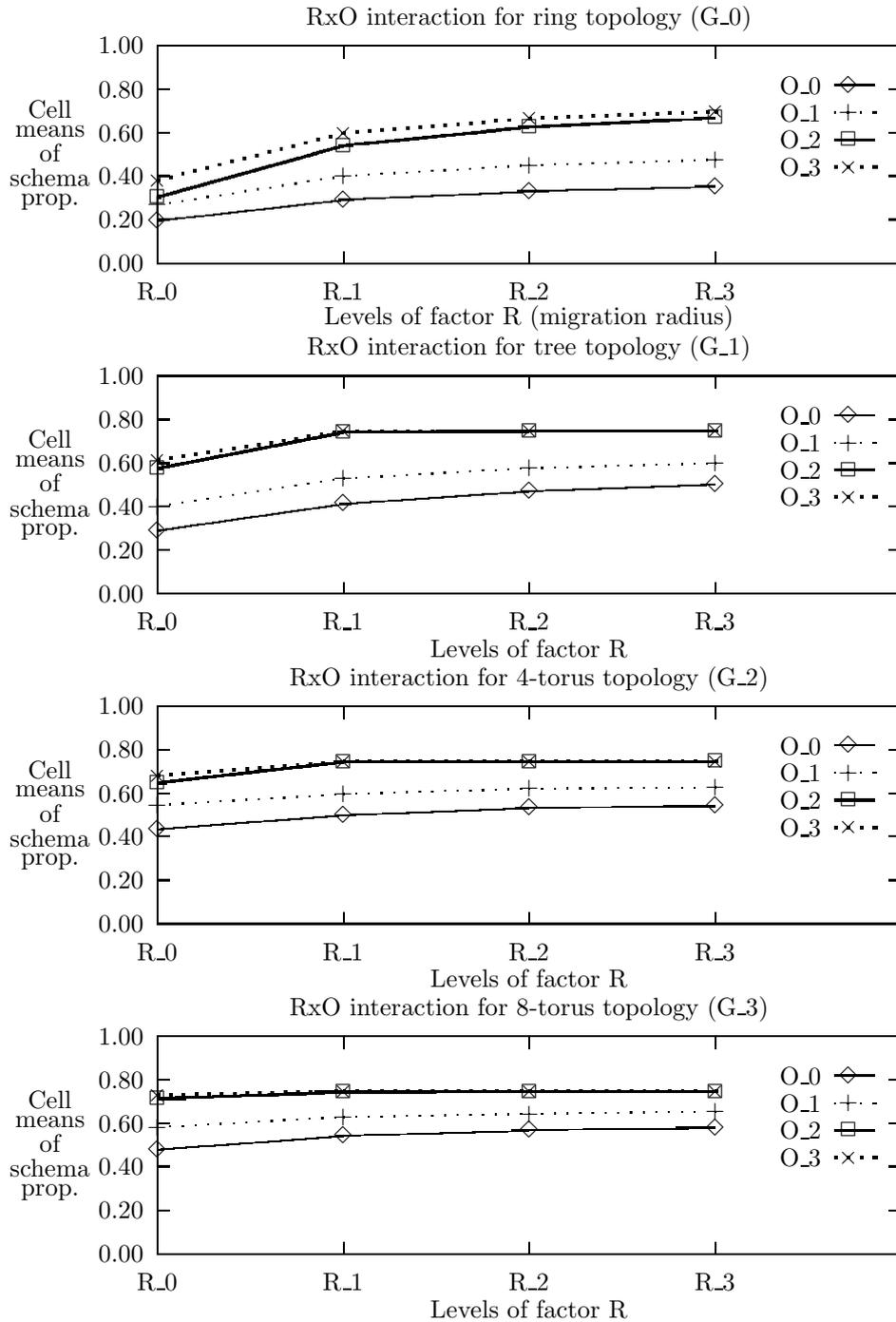


FIG. 5.7. Partial effect of  $RxO \times G$  interaction on schemata propagation

**6. Conclusions.** The motivation of this study was to rigorously but empirically examine the behavior of GAs in a SIMD environment. Such an environment imposes constraints on the parallelization including uniformity of representation, commonality of operators, and synchronous execution. With this objective in mind, a reasonable set of control parameters and response metrics were devised and a rigorous randomized experiment was performed. Approaching an empirical study from a statistical experiment design is highly effective with respect to the objectives of this study. One purpose of this paper is to demonstrate the methodology.

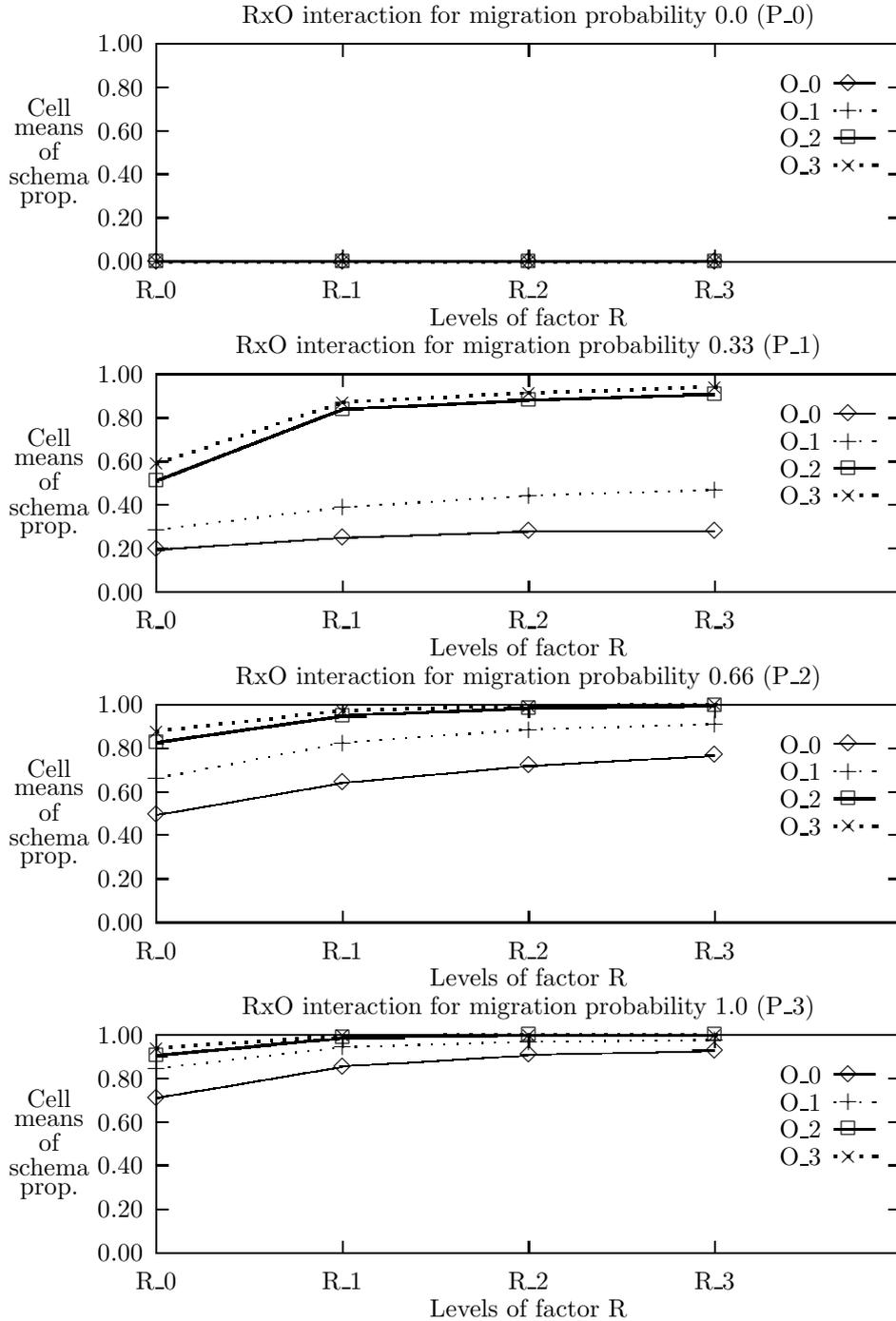


FIG. 5.8. Partial effect of RxOxP interaction on schemata propagation

The results demonstrate that migration probability and the choice of migration operator have the greatest impact. This applies to both individual and interaction effects. With respect to the migration operator, the import policy (import random vs. import best) appears more important than the replacement policy. Concerning the other factors, (topology and migration radius), the real effect is due to the degree of connectivity and both factors contribute to this implicit factor. Looking beyond the individual factors and at the degree of connectivity, it seems that above moderate levels, additional increases in connectivity have little additional effect. Finally, the

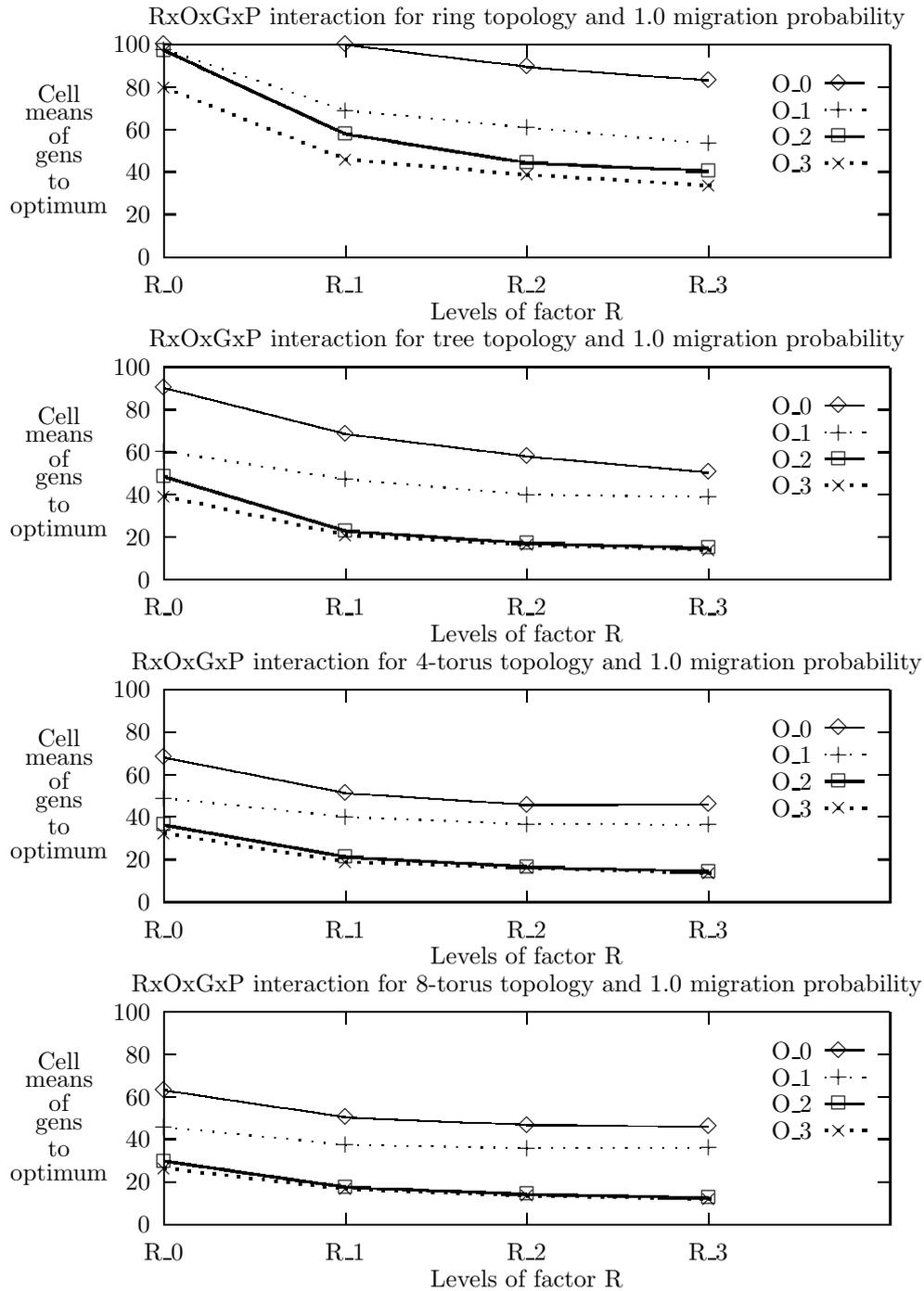


FIG. 5.9. Partial effect of  $RxOxGxP$  interaction on search efficiency.

results lend strong credence to a conclusion already largely accepted: There is a distinct behavioral difference between isolated subpopulations and those that communicate.

There are issues that are open to further study. Selection radius is one of them. Varying the selection radius has the effect of controlling the number of breeding pools to which an individual can belong. The migration operator, treated here as a single factor, deserves more detailed analysis. It consists of an import policy and a replacement policy. Each was instantiated using two options in the experiment. Other options are possible and the policies could be treated as stand-alone control parameters. Additionally, one might consider an export

policy. On the other side of the ledger, the choice of topology does not seem to be an important one as long as the communication is bidirectional. The native topologies of SIMD architectures, such as a 4- or 8-torus, can be used without impacting the performance.

**Acknowledgement.** The authors wish to thank Dr. Janet Rice of the Tulane School of Public Health for invaluable guidance in both design of the experiment and interpretation of the results. This work was supported in part by a grant from Naval Oceanographic and Atmospheric Research Laboratory, Stennis Space Center, MS, Grant # N00014-89-J-6003, in part by the NASA High Performance Computing and Communications Program (Earth and Space Sciences Project) Grant NAG 5-2216, and in part by EPSCoR grant NSF/LEQSF-ADP-04.

## REFERENCES

- [1] E. ALBA AND C. COTTA, *Evolution of complex data structures*, Informatica y Automatica, 30 (1997), pp. 42–60.
- [2] E. ALBA AND J. M. TROYA, *A survey of parallel distributed genetic algorithms*, Complexity, 4 (1999), pp. 31–52.
- [3] S. L. AMD W. PUNCH AND E. GOODMAN, *Coarse-grain parallel genetic algorithms: Categorization and new approach*, in 6th IEEE Symp. On Parallel and Distributed Processing, IEEE Press, 1994, pp. 28–37.
- [4] R. BELEW AND L. BOOKER, eds., *Proc. of Fourth Intern. Conf. on Genetic Algorithms*, San Diego, CA, April 1991, Morgan Kaufmann.
- [5] E. CANTÚ-PAZ, *A survey of parallel genetic algorithms*, Calculateurs Paralleles, Reseaux et Systems Repartis, 10 (1998), pp. 141–171.
- [6] ———, *Efficient and Accurate Parallel Genetic Algorithms*, Kluwer Academic Publishers, 2000.
- [7] ———, *Migration policies, selection pressure, and parallel evolutionary algorithms*, Journal of Heuristics, 7 (2001), pp. 311–334.
- [8] E. CANTÚ-PAZ AND D. GOLDBERG, *On the scalability of parallel genetic algorithms*, Evolutionary Computation, 7 (1999), pp. 429–449.
- [9] M. CAPCARRERE, M. TOMASSINI, A. TETTAMANZI, AND M. SIPPER, *A statistical study of a class of cellular evolutionary algorithms*, Evolutionary Computation, 7 (1999), pp. 255–274.
- [10] Y. CENSOR AND S. ZENOIS, *Parallel Optimization: Theory, Algorithms and Applications*, Oxford University Press, 1998.
- [11] ———, *Parallel algorithms in optimization*, in Handbook of Applied Optimization, M. Resende and P. Pardalos, eds., Oxford University Press, 2002, pp. 544–559.
- [12] J. P. COHOON, S. U. HEDGE, W. M. MARTIN, AND D. RICHARDS, *Punctuated equilibria: a parallel genetic algorithm*, in Grefenstette [19], pp. 148–154.
- [13] R. J. COLLINS, *Studies in Artificial Evolution*, PhD thesis, University of California, Los Angeles, 1992.
- [14] R. J. COLLINS AND D. R. JEFFERSON, *Selection in massively parallel genetic algorithms*, in Belew and Booker [4], pp. 249–256.
- [15] G. FOLINO, C. PIZZUTI, AND G. SPEZZANO, *A parallel hybrid method for SAT that couples genetic algorithms and local search*, IEEE Trans. on Evolutionary Computation, 5 (2001), pp. 323–333.
- [16] S. FORREST, ed., *Proc. of Fifth Intern. Conf. on Genetic Algorithms*, Urbana, IL, 1993.
- [17] S. FORREST AND M. MITCHELL, *The performance of genetic algorithms on Walsh polynomials: Some anomalous results and their explanation*, in Belew and Booker [4], pp. 182–189.
- [18] M. GORGES-SCHLEUTER, *An analysis of local selection in evolution strategies*, in Proc. of Genetic and Evolutionary Computation Conf., Morgan Kaufmann, 1999, pp. 847–854.
- [19] J. J. GREFENSTETTE, ed., *Proc. of Second Intern. Conf. on Genetic Algorithms*, Lawrence Erlbaum Associates, 1987.
- [20] G. HARIK, E. CANTÚ-PAZ, D. GOLDBERG, AND R. MILLER, *The gambler’s ruin problem genetic algorithms and the sizing of populations*, Evolutionary Computation, 7 (1999), pp. 231–253.
- [21] C. R. HICKS, *Fundamental Concepts in the Design of Experiments*, Holt, Rinehart and Winston, 1973.
- [22] J. H. HOLLAND, *Adaptation in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor MI, 1975.
- [23] T. ICHIMURA AND Y. KURIYAMA, *Learning of Neural Networks with Parallel Hybrid GA Using a Royal Road Function*, IEEE International Joint Conference on Neural Networks, 1998, pp. 1131–1136.
- [24] T. JONES, *A description of Holland’s Royal Road Function*, Evolutionary Computation, 2 (1994), pp. 409–415.
- [25] J. LIENIG, *A parallel genetic algorithm for performance-driven VLSI routing*, IEEE Trans. On Evolutionary Computation, 1 (1997), pp. 329–339.
- [26] B. MANDERICK AND P. SPIESSENS, *Fine-grained parallel genetic algorithms*, in Schaffer [38], pp. 428–433.
- [27] W. MARTIN, J. LIENIG, AND J. COHOON, *Island (migration) models*, in Handbook of Evolutionary Computation, T. Bäck, D. Fogel, and Z. Michalewicz, eds., Oxford University Press, 1997, pp. C6:5–8–15.
- [28] M. MITCHELL, S. FORREST, AND J. HOLLAND, *The Royal Road for genetic algorithms: Fitness landscapes and GA performance*, in Proc. 1st European Conf. on Artificial Life, MIT Press, 1992, pp. 245–254.
- [29] D. C. MONTGOMERY, *Design and Analysis of Experiments*, John Wiley & Sons, 1996.
- [30] H. MÜHLENBEIN, *Parallel genetic algorithms, population genetics and combinatorial optimization*, in Schaffer [38], pp. 416–421.
- [31] H. MÜHLENBEIN, M. SCHOMICH, AND J. BORN, *The parallel genetic algorithm as function optimizer*, Parallel Computing, 17 (1992), pp. 619–632.
- [32] M. OUSSAIDENE, B. CHOPARD, O. PICTET, AND M. TOMASSINI, *Parallel genetic programming and its application to trading model induction*, Parallel Computing, 23 (1997), pp. 1183–1198.
- [33] C. PETTEY, *Population structures: diffusion (cellular) models*, in Handbook of Evolutionary Computation, T. Bäck, D. Fogel, and Z. Michalewicz, eds., Oxford University Press, 1997, pp. C6:4–1–6.
- [34] C. PETTEY, M. LEUZE, AND J. GREFENSTETTE, *A parallel genetic algorithm*, in Grefenstette [19], pp. 155–161.

- [35] D. PRABHU, *A study in massively parallel genetic algorithms with application to image interpretation*, Tech. Rep. Tech Report 99-5, EECS Dept., Tulane University, New Orleans, 1999.
- [36] D. PRABHU, B. P. BUCKLES, AND F. E. PETRY, *MPGA: User's guide*, tech. rep., Department of Computer Science, Tulane University, New Orleans, 1995.
- [37] W. F. PUNCH, R. C. AVERILL, E. D. GOODMAN, S. C. LIN, AND Y. DING, *Using genetic algorithms to design laminated composite structures*, IEEE Expert, (1995), pp. 42–49.
- [38] J. D. SCHAFFER, ed., *Proc. of Third Intern. Conf. on Genetic Algorithms*, Fairfax, VA, 1989, Morgan Kaufmann.
- [39] J. D. SCHAFFER, R. A. CARUANA, L. J. ESHELMAN, AND R. DAS, *A study of control parameters affecting online performances of genetic algorithms for function optimization*, in Schaffer [38], pp. 51–60.
- [40] M. SIPPER, *Evolution of Parallel Cellular Machines: The Cellular Programming Approach*, Springer Verlag, 1997.
- [41] J. STENDER, *Parallel genetic algorithms theory and applications*, in Frontiers in Artificial Intelligence and Applications, J. Stender, ed., IOS Press, 1993.
- [42] H. SUZUKI AND H. SAWAI, *Crossover accelerates evolution in GAs with a Royal Road function*, in Proc. Genetic and Evolutionary Computation Conference, 2001, pp. 405–412.
- [43] R. TANESE, *Parallel genetic algorithms for a hypercube*, in Grefenstette [19], pp. 177–183.
- [44] ———, *Distributed genetic algorithms*, in Schaffer [38], pp. 434–440.
- [45] M. TOMASSINI, *Parallel and distributed evolutionary algorithms: A review*, in Evolutionary Algorithms in Engineering and Computer Science, K. Miettien, M. Mäkeä, P. Neittaanmäki, and J. Périaux, eds., J. Wiley and Sons, 1999, pp. 113–133.
- [46] S. TSUTSU AND Y. FUJIMOTO, *Forking genetic algorithm with blocking and shrinking modes (fGA)*, in Forrest [16], pp. 206–213.
- [47] D. WHITLEY, *Cellular genetic algorithms*, in Forrest [16], pp. 658–666.
- [48] D. WHITLEY, S. RANA, AND R. HECKENDORN, *Exploiting seperability in search: The island model genetic algorithm*, Journal of Computing and Information Technology, 7 (1999), pp. 33–47.
- [49] S. WONG, C. WONG, AND C. TONG, *A parallelized genetic algorithm for the calibration of the Lowry Model*, Parallel Computing, 27 (2001), pp. 1523–1536.
- [50] S. YANG, *Primal-dual genetic algorithms for Royal Road functions*, in Proc. of the 15th IFAC World Congress, Barcelona, Spain, 21-26 July 2002, pp. 439–446.

## Appendix A.

Analysis of Variance (ANOVA) for various experimental designs is adequately covered in numerous texts (e.g. [21, 29]). The experiment design used here is known as *randomized factorial*. There are four factors and four levels of each, thus  $4^4$  level combinations. For each level of each factor,  $g, r, o, p$ , the outcome of a single repetition,  $k$ , is represented by  $y_{gropk}$ . For this experiment,  $g \in \{1, 2, 3, 4\}$ ,  $r \in \{1, 2, 3, 4\}$ , and  $o \in \{1, 2, 3, 4\}$ , and  $p \in \{1, 2, 3, 4\}$ , and  $k \in \{1, 2, \dots, 10\}$ , yielding  $10 \times 4^4$  individual trials. The *degrees of freedom* are (one less) than than the number of choices possible for a component. For total trials (line 17 in Tables A.1-A.3) there are 2,559 ( $10 \times 4^4 - 1$ ) degrees of freedom. For any given factor (lines 1–4 in Tables A.1–A.3) there are three (four levels – 1) degrees of freedom. For combinations of factors, the degrees of freedom are the product of the degrees for individual factors (*i. e.*, nine for pairs, 27 for triples, and 81 for the combination of all four factors).

TABLE A.1  
ANOVA for global allelic diversity

| #  | Source                         | DF   | Sum Sq   | Mean Sq | Computed $F$ |
|----|--------------------------------|------|----------|---------|--------------|
| 1  | $G$                            | 3    | 4.2971   | 1.4324  | 45141.8887   |
| 2  | $R$                            | 3    | 3.8493   | 1.2831  | 40437.7601   |
| 3  | $O$                            | 3    | 14.6118  | 4.8706  | 153500.4667  |
| 4  | $P$                            | 3    | 81.5102  | 27.1701 | 856282.9984  |
| 5  | $G \times R$                   | 9    | 0.6086   | 0.0676  | 2131.1502    |
| 6  | $G \times O$                   | 9    | 0.9484   | 0.1054  | 3321.1950    |
| 7  | $G \times P$                   | 9    | 1.5872   | 0.1764  | 5558.0741    |
| 8  | $R \times O$                   | 9    | 0.4826   | 0.0536  | 1689.9835    |
| 9  | $R \times P$                   | 9    | 1.6751   | 0.1861  | 5865.8867    |
| 10 | $O \times P$                   | 9    | 7.1838   | 0.7982  | 25155.7570   |
| 11 | $G \times R \times O$          | 27   | 0.1644   | 0.0061  | 191.8747     |
| 12 | $G \times R \times P$          | 27   | 0.4341   | 0.0161  | 506.6511     |
| 13 | $G \times O \times P$          | 27   | 1.0641   | 0.0394  | 1242.1203    |
| 14 | $R \times O \times P$          | 27   | 0.8969   | 0.0332  | 1046.8983    |
| 15 | $G \times R \times O \times P$ | 81   | 0.3613   | 0.0045  | 140.5775     |
| 16 | Errors                         | 2304 | 0.0731   | 0.0000  |              |
| 17 | Totals                         | 2559 | 119.7481 |         |              |

TABLE A.2  
ANOVA for schemata propagation

| #  | Source                                    | DF   | Sum Sq   | Mean Sq  | Computed <i>F</i> |
|----|---|------|----------|----------|-------------------|
| 1  | <i>G</i>                                  | 3    | 16.6509  | 5.5503   | 25888.3304        |
| 2  | <i>R</i>                                  | 3    | 8.4362   | 2.8121   | 13116.4494        |
| 3  | <i>O</i>                                  | 3    | 27.5620  | 9.1873   | 42852.7267        |
| 4  | <i>P</i>                                  | 3    | 340.6299 | 113.5433 | 529602.2027       |
| 5  | <i>G</i> × <i>R</i>                       | 9    | 2.4828   | 0.2759   | 1286.7461         |
| 6  | <i>G</i> × <i>O</i>                       | 9    | 0.7596   | 0.0844   | 393.6835          |
| 7  | <i>G</i> × <i>P</i>                       | 9    | 6.8043   | 0.7560   | 3526.3902         |
| 8  | <i>R</i> × <i>O</i>                       | 9    | 0.1485   | 0.0165   | 76.9467           |
| 9  | <i>R</i> × <i>P</i>                       | 9    | 3.6515   | 0.4057   | 1892.4426         |
| 10 | <i>O</i> × <i>P</i>                       | 9    | 22.7254  | 2.5250   | 11777.6365        |
| 11 | <i>G</i> × <i>R</i> × <i>O</i>            | 27   | 0.8185   | 0.0303   | 141.4041          |
| 12 | <i>G</i> × <i>R</i> × <i>P</i>            | 27   | 1.5875   | 0.0588   | 274.2432          |
| 13 | <i>G</i> × <i>O</i> × <i>P</i>            | 27   | 3.7258   | 0.1380   | 643.6473          |
| 14 | <i>R</i> × <i>O</i> × <i>P</i>            | 27   | 2.2732   | 0.0842   | 392.7038          |
| 15 | <i>G</i> × <i>R</i> × <i>O</i> × <i>P</i> | 81   | 1.7956   | 0.0222   | 103.3959          |
| 16 | Errors                                    | 2304 | 0.4940   | 0.0002   |                   |
| 17 | Totals                                    | 2559 | 440.5459 |          |                   |

TABLE A.3  
ANOVA for search efficiency

| #  | Source                                    | DF   | Sum Sq       | Mean Sq     | Computed <i>F</i> |
|----|---|------|--------------|-------------|-------------------|
| 1  | <i>G</i>                                  | 3    | 254008.7824  | 84669.5941  | 9667.5576         |
| 2  | <i>R</i>                                  | 3    | 149389.6855  | 49796.5618  | 5685.7616         |
| 3  | <i>O</i>                                  | 3    | 579318.1168  | 193106.0389 | 22048.8096        |
| 4  | <i>P</i>                                  | 3    | 1124711.5105 | 374903.8368 | 42806.4464        |
| 5  | <i>G</i> × <i>R</i>                       | 9    | 7361.0035    | 817.8893    | 93.3864           |
| 6  | <i>G</i> × <i>O</i>                       | 9    | 46817.6848   | 5201.9650   | 593.9593          |
| 7  | <i>G</i> × <i>P</i>                       | 9    | 89067.4410   | 9896.3823   | 1129.9670         |
| 8  | <i>R</i> × <i>O</i>                       | 9    | 42775.2691   | 4752.8077   | 542.6746          |
| 9  | <i>R</i> × <i>P</i>                       | 9    | 50002.2004   | 5555.8000   | 634.3602          |
| 10 | <i>O</i> × <i>P</i>                       | 9    | 242684.8566  | 26964.9841  | 3078.8566         |
| 11 | <i>G</i> × <i>R</i> × <i>O</i>            | 27   | 3590.1730    | 132.9694    | 15.1824           |
| 12 | <i>G</i> × <i>R</i> × <i>P</i>            | 27   | 25966.8418   | 961.7349    | 109.8107          |
| 13 | <i>G</i> × <i>O</i> × <i>P</i>            | 27   | 68730.3230   | 2545.5675   | 290.6524          |
| 14 | <i>R</i> × <i>O</i> × <i>P</i>            | 27   | 41988.8262   | 1555.1417   | 177.5658          |
| 15 | <i>G</i> × <i>R</i> × <i>O</i> × <i>P</i> | 81   | 34339.4254   | 423.9435    | 48.4058           |
| 16 | Errors                                    | 2304 | 20178.7000   | 8.7581      |                   |
| 17 | Totals                                    | 2559 | 2780930.8402 |             |                   |

The fundamental idea in design of experiments is that if a factor has no significance, then the distribution of all responses is the same as the distribution of responses for a given level of the factor. That is, we could check the total mean against the mean for a specific factor level. Parametric comparisons in statistics more frequently employ an equivalent method: Determine if there are differences in the variances (mean squares) among factor levels. Let  $\bar{y}_{g,\dots}$  be the average of responses for a particular option, *g*, for the topology across all other factors and repetitions. Similarly, let  $\bar{y}_{\dots}$  be the overall average for all (in this case, 2,560) responses. To describe via example, the sum of squares for line 1 of Tables A.1-A.3 is

$$\sum_g n(\bar{y}_{g,\dots} - \bar{y}_{\dots})^2$$

where  $n$  is the number of replications (10 in our case). The mean square is the sum of squares divided by the degrees of freedom. The total sum of squares is  $\sum_g \sum_r \sum_o \sum_p \sum_p (y_{gropk} - \bar{y})^2$ . The “easy” way to compute the error sum of squares is subtract the individual factor sums from the total sum.

Variances are compared by dividing the factor mean square by the error mean square (the  $F$ -statistic). Standard tabulations of critical  $F$ -statistic values can then be consulted to determine if the effect of a factor or an interaction on the given response variable is significant. ANOVA results for the response variables measuring population diversity, schemata propagation, and search efficiency are shown in Tables A.1, A.2, and A.3, respectively.

*Edited by:* Marcin Paprzycki.

*Received:* February 10, 2003.

*Accepted:* June 16, 2003.